

## Population

Let  $X$  be a random variable  
with probability density

$$X \sim f(x)$$

$$E(X) = \mu$$

$$\text{Var}(X) = \sigma^2 = E[(X-\mu)^2]$$

Take a random sample from the  
Population of  $X$  - size  $n$

$$x_1 \ x_2 \ \dots \ x_n$$

1 realization from Prob of Sample.

STATISTIC - is a function of  
The sample - contains no  
unknown parameters.

Statistic is Random Variable.  
So, statistics have a Prob Dist

F

Use the sample to construct  
an estimate of  $\mu$ . - population mean.

MOM method-of-Moments principle.

Use sample moments to estimate  
population moments

Moments

Pop

$$E(X) = \mu$$

sample

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$$

$\bar{X}$  is the MOM estimate of  $\mu$ .

Since  $\bar{X}$  is a function of the sample (a collection of L.V.) Then it has a Sampling distribution. It is this dist. that tells us what properties  $\bar{X}$  has. and allows us to make inferences about  $\mu$  using  $\bar{X}$ .

$$E(\bar{X}) = E\left(\sum_{i=1}^n \bar{x}_i / n\right)$$

$$= \cancel{\frac{1}{n}} \cancel{\sum_{i=1}^n \bar{x}_i}$$

$$= \frac{1}{n} E\left[\sum_{i=1}^n \bar{x}_i\right]$$

$$= \frac{1}{n} [E(\bar{x}_1) + E(\bar{x}_2) + \dots + E(\bar{x}_n)]$$

if  $\bar{x}_i$  come from The population  $X$

then  $E(\bar{x}_i) = \mu$ .

$$\begin{aligned} E(\bar{X}) &= \frac{1}{n} [\mu + \mu + \dots + \mu] \\ &= \frac{n \cdot \mu}{n} = \mu. \end{aligned}$$

$$\text{Var}(\bar{X}) = \text{Var}\left(\sum_{i=1}^n \bar{x}_i / n\right)$$

$$= \frac{1}{n^2} \text{Var} \sum_{i=1}^n (\bar{x}_i)$$

if The sample is Random, Then  
 $\text{Cov}(\bar{x}_i, \bar{x}_j) = 0 \quad i \neq j$

Asgm<sup>p</sup> + t, c Theorems.

Approximations on Based on  
to make inference. Then

we use Approximate distributions  
as normal prob dist. Then  
distributed, ~~the~~ a lot in  
form of sampling

size, n.

of  $\bar{x}$ . True for large sample.

This is the exact prob. Dist

$$\bar{x} \sim \mathcal{N}(\mu, \sigma^2/n)$$

$$\text{true } \bar{x} \sim \mathcal{N}(\mu, \sigma^2/n)$$

$$\bar{x} \sim (\mu, \sigma^2/n)$$

$$n/\sigma^2 = (x - \bar{x})^2 \cdot T^{-1} =$$

$$\text{Var}(\bar{x}) = \frac{1}{T} \left[ \text{Var}(x_1) + \text{Var}(x_2) + \dots + \text{Var}(x_n) \right] =$$

Two Basic Asymptotic Theorems  
used in STAT.

(i) Law of Large Numbers

This 'Law' says that Sample Means converge in Probability to population means.

$$\bar{X} \xrightarrow{P} \mu$$

Convergence in Probability -

As the sample gets larger,  
the probability that  $\bar{X}$  gets 'close' to  $\mu$  converges to 1.

$$\lim_{n \rightarrow \infty} \Pr(|\bar{X} - \mu| < \varepsilon) = 1$$

$\varepsilon > 0$  any ~~non~~ pos.  $\varepsilon$ .

"Probability limit"

If a statistic converges in probability to some number, then we say that

The statistic is a consistent estimator of that number (or parameter)

The CLN say that the sample mean is a consistent estimator of the population mean.

The various CLN Theorems tell you the conditions under which  $\bar{X}$  is consistent for  $\mu$ .

e.g.,  $X$  has finite mean and variance. If so, then

$$\bar{X} \xrightarrow{P} E(X) = \mu.$$

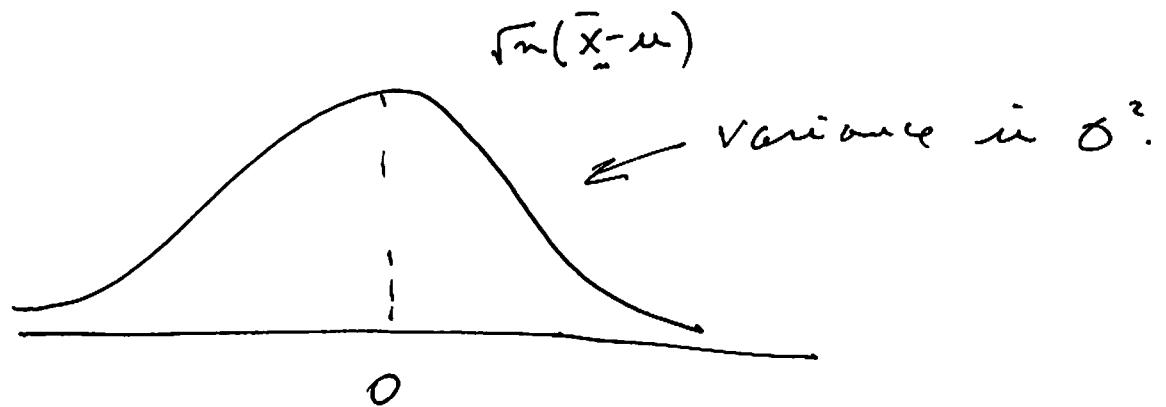
The other main Theorem (or set of Theorems) ~~is~~ is The Central Limit Theorem.

They say that under certain circumstances,  $\bar{X}$  will be approximately normally dist. if  $n$  is very large.

$$X \sim (\mu, \sigma^2)$$

take sample of size,  $n$ .  
let  $n \rightarrow \infty$

$$\sqrt{n}(\bar{X} - \mu) \xrightarrow{d} N(0, \sigma^2)$$



Then, in finite samples

$$\sqrt{n}(\bar{x} - \mu) \stackrel{a}{\sim} N(0, \sigma^2)$$

$$(\bar{x} - \mu) \stackrel{a}{\sim} N\left(\frac{\mu}{\sqrt{n}}, \frac{\sigma^2}{n}\right)$$

$$\bar{x} \stackrel{a}{\sim} N\left(\mu, \frac{\sigma^2}{n}\right)$$

if  $n$  is "Big" enough.