

Functional Form

Thus far, we've been using the CLRM to estimate lines. What if the relationship between y & x is not linear.

It turns out that linear regression can be used to estimate all sorts of shapes - very flexible.

linear does not mean a linear relationship between variables; but it refers to a model that is linear in the parameters. The implication of this is that we can transform the variables using any function we like as long as the model is linear in β 's.

Example:

$$y = \beta_1 + \beta_2 x + e$$

$$y = \beta_1 + \beta_2 x^2 + e$$

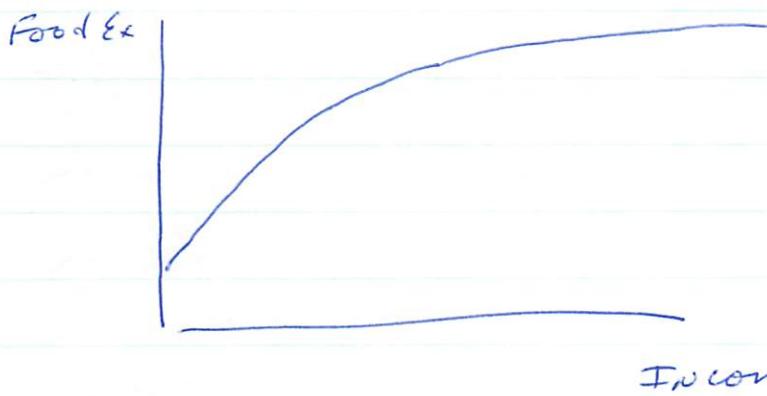
$$\ln(y) = \beta_1 + \beta_2 \ln(x) + e.$$

Excluded $y = \beta_1 + \frac{x^{\beta_2}}{\ln(\beta_2)} + e$.

allowing $\beta_1 x^{\beta_2} + e$

$$y = \beta_1 + \beta_2 x^{\beta_2} + e$$
 can be estimated

In the Food Exp example, it is unlikely that Food Exp is linearly related to Income more likely



How could we use linear regression to estimate these relationships.

Electricity

Elasticities measured $\frac{\% \Delta y}{\% \Delta x}$

for continuous functions there is

$$\frac{dy}{dx} \cdot \epsilon_{yx} = \frac{\Delta y / y}{\Delta x / x} = \frac{\Delta y}{\Delta x} \cdot \frac{x}{y}$$

For continuous functions $\frac{dy}{dx} = \frac{dy}{dx}$

$$\therefore \epsilon_{yx} = \frac{dy}{dx} - \frac{x}{y}$$

In The Linear Regression model

$$y_t = \beta_1 + \beta_2 x_t + e_t$$

$$\frac{\partial y}{\partial x} = \beta_2 \quad \text{slope}$$

$$\frac{\partial y}{\partial x} \cdot \frac{x}{y} = \beta_2 \frac{x}{y} \quad \text{Elasticity}$$

here it depends on "where you are".

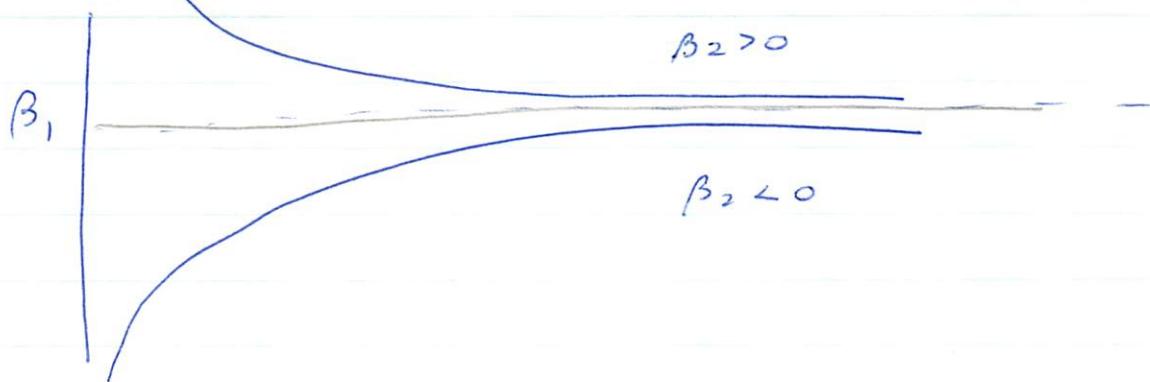
Commonly Used functional Forms : slopes & elast.

Reciprocal model.

$$y_t = \beta_1 + \beta_2 \frac{1}{x_t} + e_t$$

$$\frac{dy}{dx} = -\beta_2 \quad \text{slope} \quad -\beta_2 \frac{1}{x_t^2}$$

$$\text{Elast} \quad -\beta_2 \frac{1}{x_t} y_t$$



log-log model: constant elasticity model.

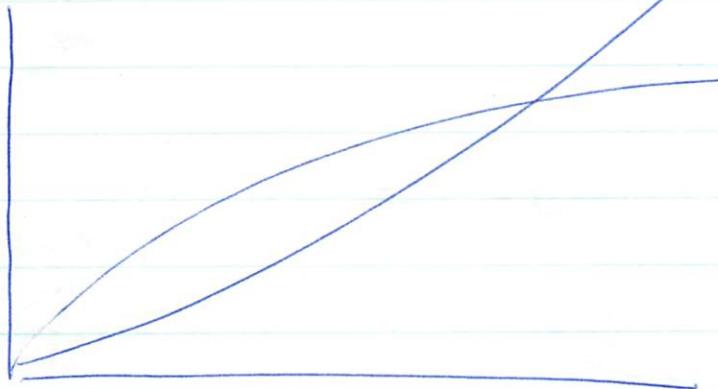
$$\ln(y_t) = \beta_1 + \beta_2 \ln(x_t) + \epsilon_t$$

Slope $\beta_2 \frac{y_t}{x_t} \frac{y_t}{x_t}$

Elast β_2

$$\beta_2 > 1$$

$$0 < \beta_2 < 1$$



$$y_t = e^{\beta_1 + \beta_2 \ln x_t + \epsilon_t}$$

$$e^{-\infty} = 0$$

$$\beta_2 > -1 \quad \beta_2 < -1$$

$$\beta_2 > -1$$

$$0 < \beta_2 < -1$$

$$\beta_2 = -1$$

$$\beta_-$$



Functional Forms

Log-linear $\ln(y) = \beta_1 + \beta_2 x + \epsilon$

Linear log $y = \beta_1 + \beta_2 \ln(x) + \epsilon$

log-log $\ln(y) = \beta_1 + \beta_2 \ln(x) + \epsilon$

Log-linear

$$\ln(y) = \beta_1 + \beta_2 x + \epsilon.$$

$$E(\ln(y)) = \beta_1 + \beta_2 x$$

$$\text{if } E(\epsilon|x) = 0$$

For now, we'll just ignore the errors.

$$y = e^{(\beta_1 + \beta_2 x)}$$

Interpretation of β_2 .

$$\frac{d \ln(y)}{d x} = \beta_2$$

$$\frac{d \ln(y)}{d x} \cdot \frac{dy}{dy} = \frac{d \ln(y)}{dy} \cdot \frac{dy}{dx} = \beta_2$$

$$\frac{d \ln y}{dy} = \frac{1}{y} \quad \therefore \quad \beta_2 = \frac{dy/y}{dx} \approx \frac{\% \Delta y}{\Delta x}$$

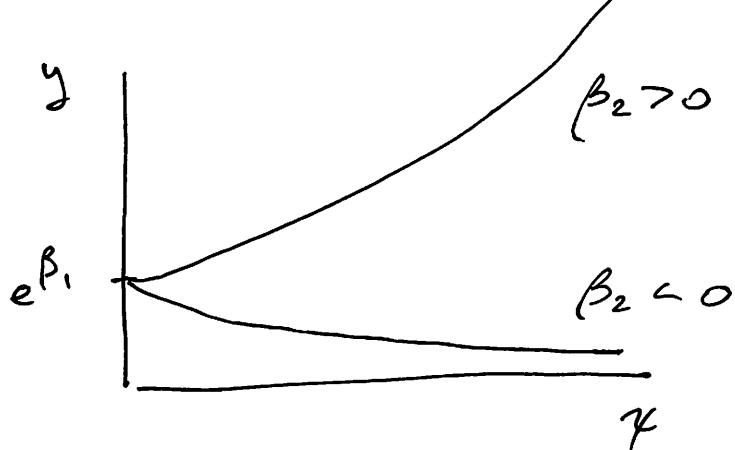
A 1 unit change in x leads
to a $\% \Delta$ in y .

Slope -

$$\frac{dy}{dx} = \beta_2 \cdot y$$

Elasticity

$$\begin{aligned} \frac{dy}{dx} \cdot \frac{x}{y} &= \beta_2 y \cdot \frac{x}{y} \\ &= \beta_2 x. \end{aligned}$$



Linear Log

$$y = \beta_1 + \beta_2 \ln(x) + \epsilon.$$

Interpretation of β_2

$$\frac{\partial y}{\partial \ln(x)} = \beta_2$$

$$\frac{\partial y}{\partial \ln(x)} \cdot \frac{\partial \ln(x)}{\partial x} = \beta_2 = \frac{\partial y}{\partial x} \cdot \frac{\partial \ln(x)}{\partial x}$$

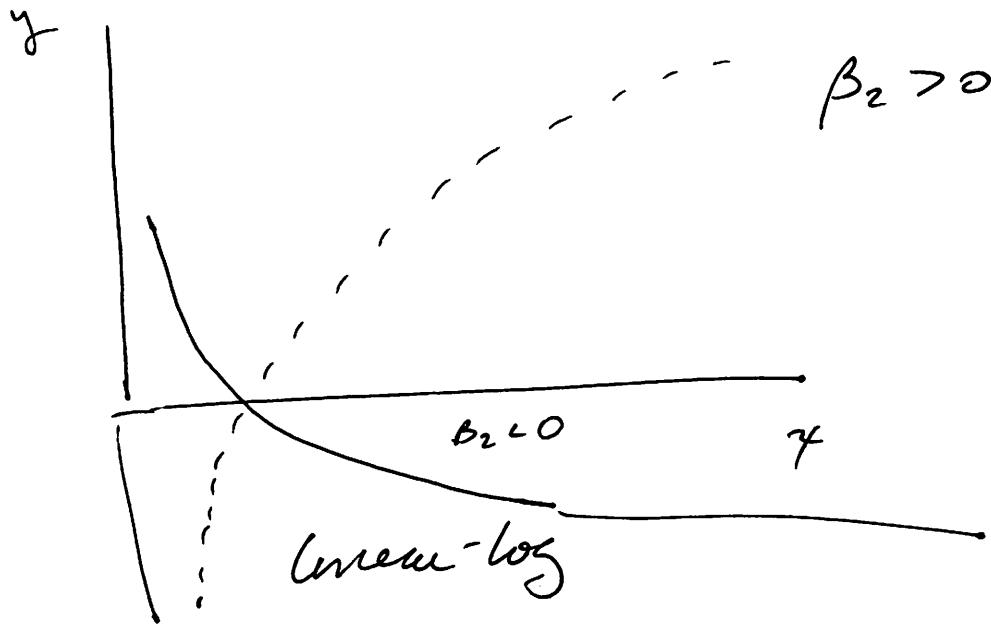
$$\frac{\partial \ln(x)}{\partial x} = \frac{1}{x} = \frac{1}{x} = x \quad \therefore$$

$$\beta_2 = \frac{\partial y}{\partial x/x} \doteq \Delta y / \% \Delta x.$$

x changes by 1% and the level
of y changes in units.

$$\text{Slope: } \frac{\partial y}{\partial x} = \beta_2/x$$

$$\text{Elasticity } \frac{\partial y}{\partial x} \cdot \frac{x}{y} = \frac{\beta_2}{x} \cdot \frac{x}{y} = \beta_2/y.$$



log-log

$$\ln(y) = \beta_1 + \beta_2 \ln(x) + \epsilon$$

$$\beta_1 + \beta_2 \ln(x) + \epsilon.$$

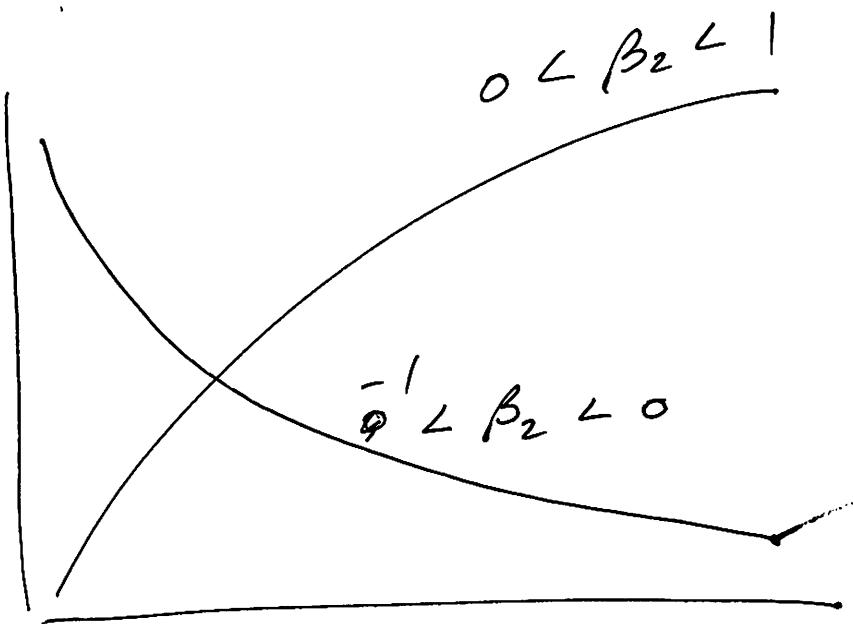
$$y = e^{\beta_1 + \beta_2 \ln(x) + \epsilon}$$

$$\frac{\partial \ln y}{\partial \ln x} = \beta_2$$

$$\frac{\partial \ln y}{\partial y} \cdot \frac{\partial y}{\partial x} \cdot \frac{\partial x}{\partial \ln x} = \beta_2 = \frac{\partial y}{\partial x} \cdot \frac{1}{y} \cdot x$$

$\beta_2 \approx \% \Delta y / \% \Delta x$ elasticity

Slope : $\frac{\partial y}{\partial x} = \beta_2 y/x$



log-inverse

$$\ln(y) = \beta_1 + \beta_2 \frac{1}{x} + u$$

$$\beta_1 + \beta_2/x + u$$

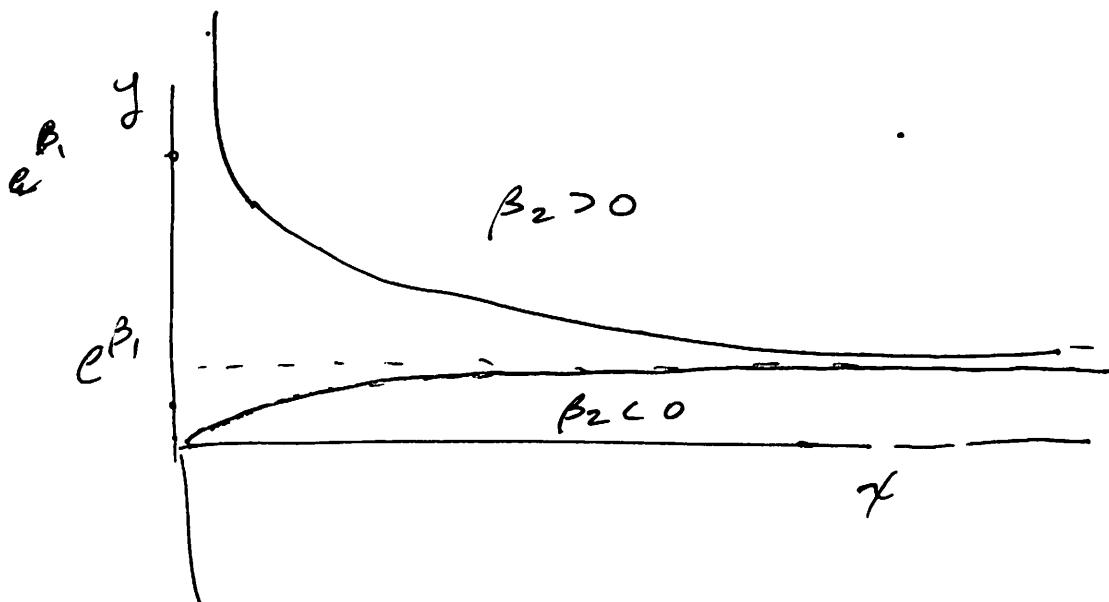
$$y = e^{\beta_1 + \beta_2/x + u}$$

$$\frac{\partial \ln y}{\partial x} = -\beta_2 x^{-2} = -\beta_2/x^2$$

$$\frac{\partial \ln y}{\partial y} \cdot \frac{\partial y}{\partial x} = \frac{1}{y} \cdot \frac{\partial y}{\partial x} = -\beta_2/x^2$$

slope: $\frac{\partial y}{\partial x} = -\beta_2 \frac{1}{x^2}$

elastic: $\frac{\partial y}{\partial x} \cdot \frac{x}{y} = -\beta_2/x$.



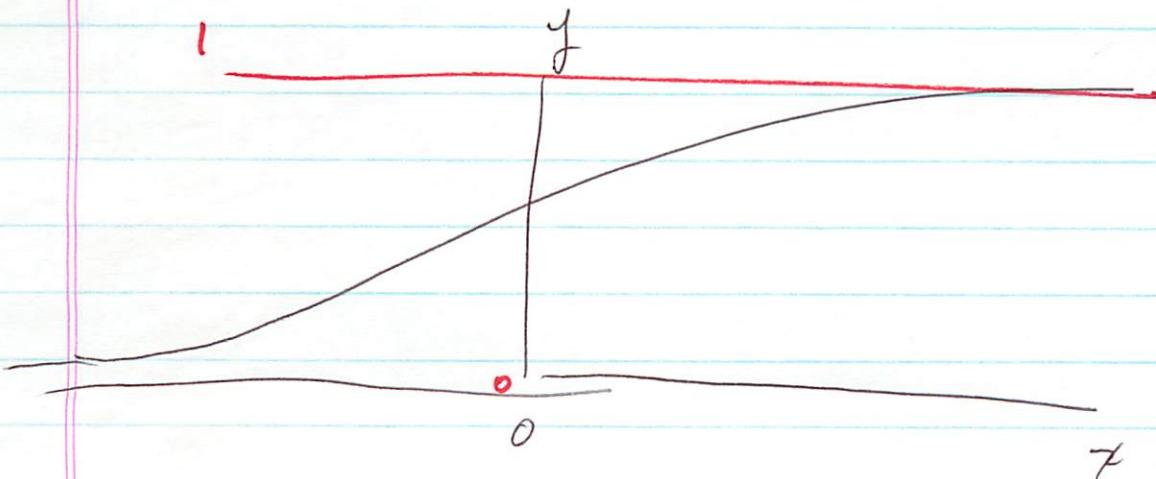
logit: suppose $0 < y < 1$

$$\ln\left(\frac{y}{1-y}\right) = \beta_0 + \beta_1 x + \epsilon$$

o $\ln\left(\frac{y}{1-y}\right)$ has

To get predictions not lie between 0 and 1 we can use logit

$$y = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x + \epsilon)}}$$



Slope: $\beta_1 y(1-y)$

n_d : $\beta_1 (1-y)x$