

Estimation

1) collect a sample of size T .

$$y_t = \beta_1 + \beta_2 x_t + e_t \quad t = 1, \dots, T$$

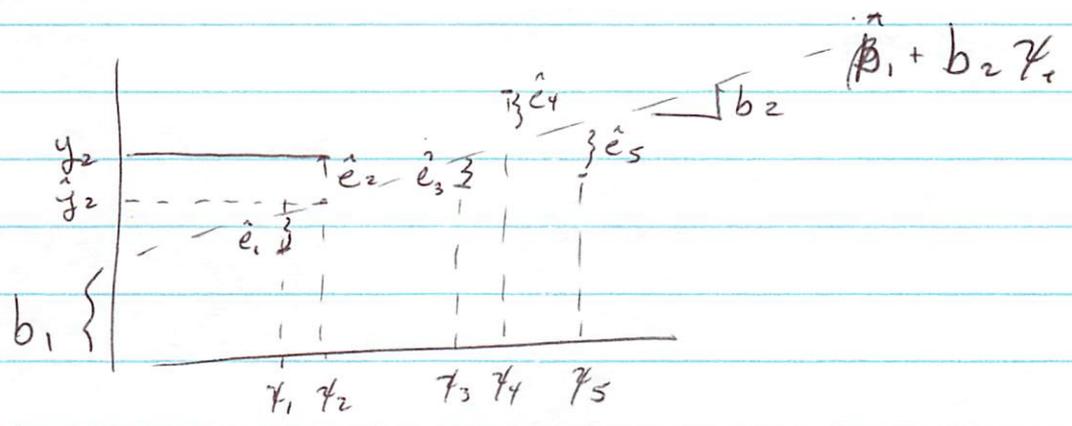
$$e_t \text{ iid } (0, \sigma^2)$$

x_t 's vary.

Least Squares

One way to estimate β_1, β_2 is to use least squares.

LS fits a line through the data that minimizes the SSE



b_1, b_2 are chosen to make $\sum_{t=1}^T e_t^2$ as small as possible.

So, it becomes a simple minimization exercise.

$$e_t = y_t - \beta_1 + \beta_2 x_t$$

$$e_t^2 = (y_t - \beta_1 + \beta_2 x_t)^2$$

$$S = \sum e_t^2 = \sum (y_t - \beta_1 + \beta_2 x_t)^2$$

$$(1)' \quad \frac{\partial S}{\partial \beta_1} = 2 \sum (y_t - \beta_1 - \beta_2 x_t) (-1)$$

$$(2)' \quad \frac{\partial S}{\partial \beta_2} = 2 \sum (y_t - \beta_1 - \beta_2 x_t) (-x_t)$$

$$(1)'' \quad = -2 \sum y_t + 2T\beta_1 + 2 \sum \beta_2 x_t$$

$$(2)'' \quad = -2 \sum y_t x_t + 2 \sum \beta_1 x_t + 2 \sum \beta_2 x_t^2$$

To minimize, we set = zero and solve for b_1, b_2

$$(1)''' \quad = 2(\sum y_t - T b_1 - \sum x_t b_2) = 0$$

$$(2)''' \quad = 2(\sum x_t y_t - \sum x_t b_1 - \sum x_t^2 b_2) = 0$$

Rearranging

$$(1) \quad T b_1 + \sum x_i b_2 = \sum y_i$$

$$(2) \quad \sum x_i b_1 + \sum x_i^2 b_2 = \sum x_i y_i$$

There are known as LS normal eq.
2 eqs in 2 unknowns, so solve for b_1, b_2

Multiply the first by $\sum x_i$ and the second by T , then subtract: (1) - (2)

$$\begin{aligned} T \sum x_i b_1 + \sum x_i \cdot \sum x_i b_2 &= \sum x_i \sum y_i \\ - T \sum x_i b_1 - T \sum x_i^2 b_2 &= -T \sum x_i y_i \end{aligned}$$

$$\left((\sum x_i)^2 b_2 - T \sum x_i^2 \right) b_2 = -T \sum x_i y_i + \sum x_i \sum y_i$$

$$b_2 = \frac{T \sum x_i y_i - \sum x_i \sum y_i}{T \sum x_i^2 - (\sum x_i)^2}$$

To get b_1 divide (1) by T

$$T b_1 + \sum x_i b_2 = \sum y_i$$

$$b_1 + \sum x_i / T b_2 = \sum y_i / T$$

$$\bar{y} - \bar{x} b_2$$

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Plugging in $f_{i,t}$ into b_1, b_2
we get estimates

b_1, b_2 are rules for using data
- any data - and are estimates.

Estimates are obtained by plugging
in a data set into the rule.

Other quantities of interest

Once you've estimated the model you could make predictions

$$\hat{y} = 40.76 + .1282x$$

$$\text{Income } \$750/\text{week} \Rightarrow \hat{y} = \$136.98$$

Elasticities

$$\frac{\% \Delta \text{ Food Exp}}{\% \Delta \text{ Income}}$$

is approximated as

$$\frac{\Delta \text{ Food Exp} / \text{Food Exp}}{\Delta \text{ Income} / \text{Income}} = \frac{\Delta \text{ Food Exp}}{\Delta \text{ Inc.}} \cdot \frac{\text{Income}}{\text{Food Exp.}}$$

$$\text{In the linear model} = \beta_2 \cdot \frac{x}{y}$$

Properties of Est. LS

What can we expect to happen if we use LS to estimate β_1, β_2 ?

* Remember, b_1, b_2 are rules for using data to obtain estimates of parameters. They are functions of R.V. (y 's) and \therefore have pdf's that can be characterized by means, variance, covariance etc.

* How do we decide whether LS is a good rule to use? are there others that are ~~is so~~ by some measure better?

Sampling Properties

b_1, b_2 are R.V. that have pdf's that can be studied prior to collection of any data (analyze them over all possible realizations of the data)