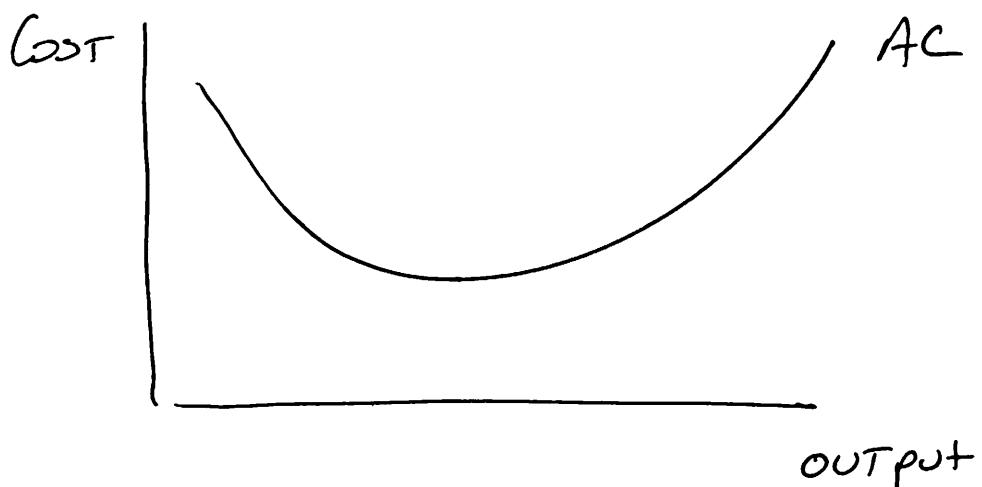


# Polynomials

Another way to introduce nonlinearity into a linear regression is through the introduction of polynomial terms.

## Cost Curves



$$AC = \beta_1 + \beta_2 Q + \beta_3 Q^2 + e$$

Adding  $Q^2$  allows function to be PARABOLIC.

Slope  $\frac{E(AC)}{\partial Q} = \beta_2 + 2\beta_3 Q$

$\beta_2 < 0$   
 $\beta_3 > 0$



$$TC = \beta_1 + \beta_2 Q + \beta_3 Q^2 + \beta_4 Q^3 + \epsilon.$$

$$MC = \frac{\partial TC}{\partial Q} = \beta_2 + 2\beta_3 Q + 3\beta_4 Q^2$$

i.e., MC is a parabola.

Very Flexible But.

Biggest Problem:  $Q, Q^2, Q^3$  highly collinear. Std errors will be large and it is possible that LS can't be computed (especially if  $Q$  is large)

$$\text{Sales} = \beta_0 + \beta_1 \text{Price} + \beta_3 A + \beta_4 A^2 + \epsilon$$

$A$  = \$1000 of Advertising.

Sales = \$1000 of Sales.

$$\frac{\partial E(\text{Sales})}{\partial A} \Big|_{\text{Price constant}} = \beta_3 + 2\beta_4 A$$

we expect  $\beta_3 > 0$ ,  $\beta_4 < 0$

diminishing returns to Ad.

Suppose you want to Advertise to the point

where ~~Advertising \$ spent~~ =

the last \$ spent on  $A$  is  
just equal to increase in  
Revenue.

$$\beta_3 + 2\beta_4 A = 1$$

MP of 1\$  $A$  = \$1 Revenue.

solve for  $A$ :

$$2 \beta_4 A_o = 1 - \beta_3$$

$$A_o = \frac{1 - \beta_3}{2 \beta_4}$$

Estimate: plug LS estimates  
in

$$\hat{A}_o = \frac{1 - \hat{\beta}_3}{2 \hat{\beta}_4} \approx 2.014$$

$\Rightarrow \$2014$  "optimal"

Variance of  $\hat{A}_o$ : The "Delta" Method

Based on a First Order Taylor Series Expansion.

$$\begin{aligned} \text{Var}(\hat{A}) &= \left( \frac{\partial A}{\partial \beta_3} \right)^2 \text{Var}(\hat{\beta}_3) + \left( \frac{\partial A}{\partial \beta_4} \right)^2 \text{Var}(\hat{\beta}_4) \\ &\quad + 2 \cdot \left( \frac{\partial A}{\partial \beta_3} \right) \left( \frac{\partial A}{\partial \beta_4} \right) \cdot \text{Cov}(\hat{\beta}_3, \hat{\beta}_4) \end{aligned}$$

$$\frac{\partial A}{\partial \beta_3} = -\frac{1}{2\beta_4} \quad \frac{\partial A}{\partial \beta_4} = \frac{1-\beta_3}{2\beta_4^2}$$

$$VAR(\hat{A}) = \left( \frac{1}{2(2.768)} \right)^2 12.646 + \left( \frac{1-12.151}{2(2.768)^2} \right) \times 88477$$

$$+ 2 \left( \frac{1}{2(2.768)} \right) \left( \frac{1-12.151}{2(2.768)^2} \right) \cdot 3.2887$$

$$= .0166$$

$$\widehat{SE} = \sqrt{.0166} \approx .1287$$

95% CI

$$2.014 \pm (1.994)(.1287)$$

$$(1.757, 2.271)$$

## Interactions

$$\text{Sales} = \beta_1 + \beta_2 \text{Price} + \beta_3 \text{Adv} + \epsilon.$$

$$\frac{\partial E(\text{Sales})}{\partial A} = \beta_3$$

Suppose we want  $\beta_3$  to depend on a variable? like Adv.

$$\beta_3 = S_1 + S_2 \text{Adv};$$

$$\begin{aligned}\text{Sales} &= \beta_1 + \beta_2 \text{Price} + (S_1 + S_2 \text{Adv}) \cdot \text{Adv} + \epsilon \\ &= \beta_1 + \beta_2 P + S_1 \text{Adv} + S_2 \text{Adv}^2 + \epsilon.\end{aligned}$$

This results in Quadratic Polynomial Model. But, The Marginal effect of A on Sales would depend on other variables.

" "   
  $\Rightarrow$  Interactions

## Pizza Model

$$\text{Pizza}_A = \beta_1 + \beta_2 \text{AGE} + \beta_3 \text{Income} + \epsilon.$$

Pizza: Annual Exp of Pizza \$1

AGE : years

Income: \$1,000

Suppose you think the ME of  
AGE depends on income.

As a person ages, he spends  
less. Also, the effect of age  
increases the more income you  
have.



$$\beta_2 = \delta_1 + \delta_2 \text{ Income}.$$

$$\delta_1 < 0, \delta_2 < 0$$

$$Pizza = \beta_1 + (\delta_1 + \delta_2 \text{ Income}) AGE$$

$$+ \beta_3 \text{ Income} + \epsilon$$

$$= \beta_1 + \delta_1 AGE + \delta_2 Inc \cdot AGE$$

$$+ \beta_3 Inc + \epsilon.$$

Inc. AGE is called an  
"Interaction" term.

$\Rightarrow$  How the ME of 1 variable  
depends on the level of  
another.

1. *Chlorophytum comosum* L. (Liliaceae) 2. *Clivia miniata* (L.) Sweet (Amaryllidaceae)

19. 10. 1962

10. *Leucosia* *leucostoma* *leucostoma* *leucostoma* *leucostoma* *leucostoma*

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27-28

W. H. BROWN, JR., M.D.

1. *Leucosia* *leucostoma* *leucostoma* *leucostoma* *leucostoma* *leucostoma*

*W. C. L. - 1903*

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