

11/5/15

Valid test is one in which
The Nominal size of the test
is ~~not~~ close to or converging
towards The actual size as
 $n \rightarrow \infty$.

Nominal size is size or significance
that user chooses.

Actual size is the proportion of
time a true H_0 is rejected \Rightarrow
By the test.

Not Valid \Rightarrow implies that The
actual distribution of test statistic
is not converging to The power distribution.

Valid hypothesis test about parameters
of a model. Required:

(1) β must have a
consistent estimator of β .

$\Rightarrow \hat{\beta}$ is a consistent for β .
estimator.
 \hookrightarrow it has a variance

or precision associated
with the particular estimator

$\text{Var}(\hat{\beta})$ - precision of $\hat{\beta}$

(2) $\hat{\beta}$ must have a consistent
estimator of $\text{Var}(\hat{\beta})$

$\Rightarrow \text{Var}(\hat{\beta})$

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Solution: Wald Test. - estimates both $\hat{\beta}_2$ and $\hat{\beta}_3$ precision

$$t = \frac{\hat{\beta}_2}{\text{std error } (\hat{\beta}_2)}$$

std error ($\hat{\beta}_2$)

$$y = \beta_1 + \beta_2 x_2 + \beta_3 x_3 + \text{error}.$$

$$H_0: \beta_2 = 0, \beta_3 = 0$$

$$H_A: \beta_2 \neq 0 \text{ or } \beta_3 \neq 0$$

$$\hat{\beta} = \begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \end{pmatrix} \quad \text{cov}(\hat{\beta}) = \begin{pmatrix} \text{var}(\hat{\beta}_1) & & \\ & \text{cov}(\hat{\beta}_1, \hat{\beta}_2) & \text{var}(\hat{\beta}_2) \\ & \text{cov}(\hat{\beta}_1, \hat{\beta}_3) & \text{cov}(\hat{\beta}_2, \hat{\beta}_3) & \text{var}(\hat{\beta}_3) \end{pmatrix}$$

$3x_1$

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$$\hat{\beta} \sim N\left(\beta, \text{cov}(\hat{\beta})\right)$$

3×1 3×3

Thm:

$$\begin{matrix} \hat{\beta} - \beta \\ 1 \times 3 \end{matrix} \sim N\left(\hat{\beta}, \text{cov}(\hat{\beta})\right) \sim \chi^2_{k=3}$$

$$\hat{\beta} \sim N(\beta, \sigma^2) \quad \frac{\hat{\beta} - \beta}{\sqrt{\sigma^2}} = N(0, 1)$$

$$\frac{(\hat{\beta} - \beta)^2}{\sigma^2} \sim \chi^2_1$$

Linear combinations of variables are normal.

$$\begin{aligned} \hat{\beta} &\sim N\left(\beta, \text{cov}(\hat{\beta})\right) \\ R\hat{\beta} &\sim N\left(R\beta, R\text{cov}(\hat{\beta})R^T\right) \end{aligned}$$

$$(R\hat{\beta} - R\beta)^T [R \text{cov}(\hat{\beta}) R^T]^{-1} (R\hat{\beta} - R\beta) \sim \chi_{J}^2 \quad (5)$$

τ under H_0

τ under H_1

R is $J \times k$ matrix of known constants

linear transformation of β .

$$H_0: \beta_2 = 0, \beta_3 = 0$$

$$R\beta = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = 0$$

R

$$\tau = 0$$

$$H_0: R\beta = \tau$$

$$\beta_2 = 0$$

$$\beta_3 = 0$$

\Leftarrow

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$$(R \hat{\beta} - r)^T \Rightarrow 1 \times 5$$

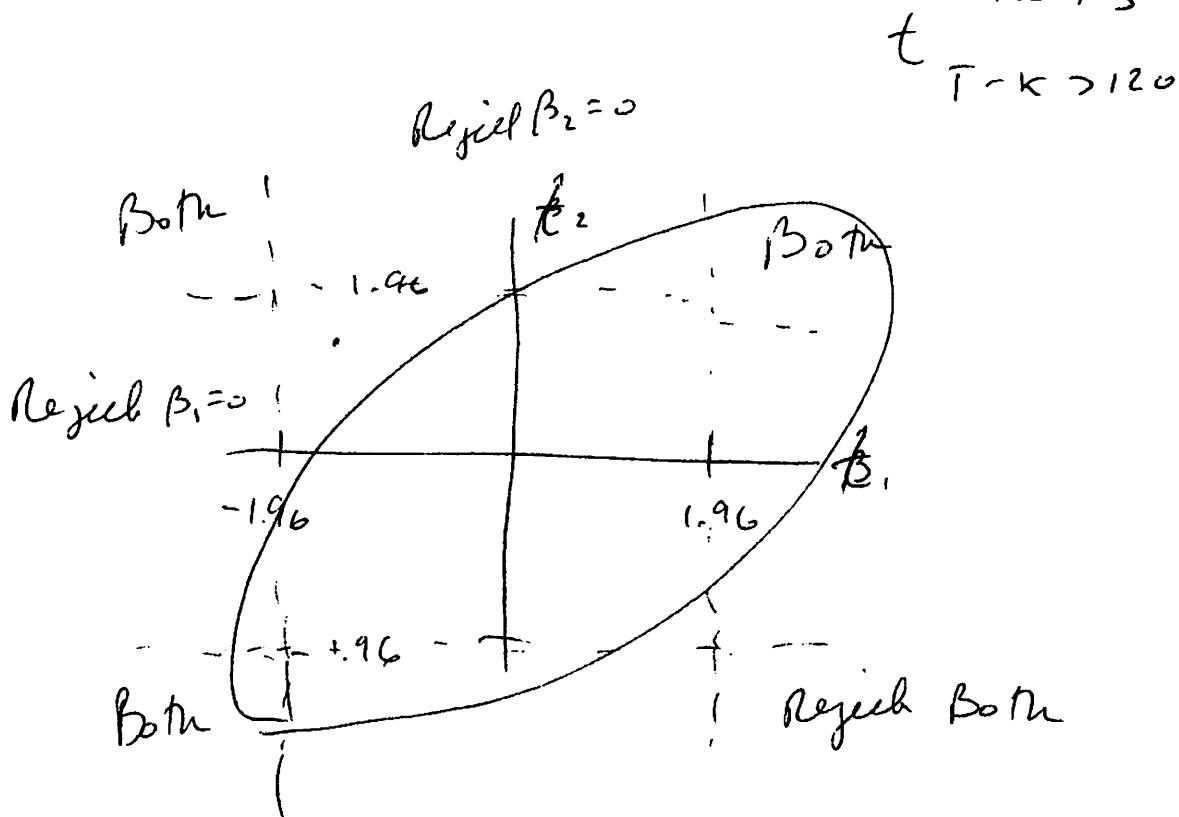
$$\Sigma_{K \cdot kx_1} - \Sigma_{x_1} \Rightarrow$$

$$W = \begin{pmatrix} 0 & \hat{\beta}_2 - 0 & \hat{\beta}_3 - 0 \end{pmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} \begin{pmatrix} 0 \\ \hat{\beta}_2 \\ \hat{\beta}_3 \end{pmatrix} \sim \mathcal{N}_2 \text{ if } \begin{cases} \hat{\beta}_2 = 0 \\ \hat{\beta}_3 = 0 \end{cases}$$

3x3

$$W = (\hat{\beta}_2)^2 c_{22} + (\hat{\beta}_3)^2 c_{33} + 2 \hat{\beta}_2 \hat{\beta}_3 c_{32} c_{23}$$

Not the same as individual t-tests



F stat is ellipse

and takes into account

$$\text{that } \text{Cov}(\hat{\beta}_1, \hat{\beta}_2) \neq 0$$

so, you would not reject \$\beta_1, \beta_2 = 0\$ by t-stats but be in reject region of joint t-test.

or in not reject region of F
but Reject Both in t-tests.