

$$\text{Elasticity} = \text{slope} \cdot \frac{\text{SQFT}}{P}$$

$$= 2\alpha_2 \cdot \text{SQFT}^2 / P$$

log linear

$$\ln(\text{Price}) = \beta_1 + \beta_2 \text{SQFT} + \epsilon_i$$

$$\frac{\partial \ln(P)}{\partial \text{SQFT}} = \frac{\partial \ln(P)}{\partial P} \cdot \frac{\partial P}{\partial \text{SQFT}} = \beta_2$$

$$= \frac{1}{P} \cdot \frac{\partial P}{\partial \text{SQFT}} = \beta_2$$

$$\therefore \frac{\partial P}{\partial \text{SQFT}} = \beta_2 \cdot \text{Price}.$$

Predictions

$$\hat{\ln(\text{Price})} = \hat{\beta}_1 + \hat{\beta}_2 \text{SQFT}$$

$$e^{\hat{\ln(\text{Price})}} = e^{\hat{\beta}_1 + \hat{\beta}_2 \text{SQFT}}$$

$$\text{Price} = e^{\hat{\beta}_1 + \hat{\beta}_2 \text{SQFT}}$$

## Sampling Variation

$$\hat{\beta}_1 \sim (\beta_1, \text{Var}(\hat{\beta}_1))$$

$$\hat{\beta}_2 \sim (\beta_2, \text{Var}(\hat{\beta}_2))$$

when  $y_i = \beta_1 + \beta_2 x_i + \epsilon_i \quad i=1, 2, \dots, N$

$\epsilon_i \text{ iid } (0, \sigma^2)$        $E(\epsilon_i | x_i) = 0$   
 Homosched.  
 no cov.

$x_i \neq \text{constant}$

$$\text{Var}(\hat{\beta}_1) = \sigma^2 \left[ \frac{\sum x_i^2}{N \sum (x_i - \bar{x})^2} \right]$$

$$\text{Var}(\hat{\beta}_2) = \sigma^2 \left[ \frac{1}{\sum (x_i - \bar{x})^2} \right]$$

$$\text{Cov}(\hat{\beta}_1, \hat{\beta}_2) = \sigma^2 \left[ \frac{-\bar{x}}{\sum (x_i - \bar{x})^2} \right]$$

↙ Var of  $y$ .

Notice:  $\text{Var}(\hat{\beta}_2) = \frac{\sigma^2}{n} \left( \frac{1}{\sum (x_i - \bar{x})^2 / n} \right)$

↙  
Sample size      ↗  
                    Sample Variance of  $x$ .

Also, Notice that there is only 1 unknown parameter. Estimating  $\sigma^2$  consistently will yield consistent estimates of  $\text{Var}(\hat{\beta}_1)$  and  $\text{Var}(\hat{\beta}_2)$ .

### Estimating $\sigma^2$

$$\text{Var}(e_i) = \sigma^2 = E[(e_i - \bar{e}_i)^2]$$

$$\text{Now } \frac{1}{n} \sum (e_i - \bar{e}_i)^2$$

Replacing  $e_i$  with consistent estimate  $\hat{e}_i = y_i - \hat{\beta}_1 - \hat{\beta}_2 x_i$

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^n \hat{e}_i^2}{n}$$

This is biased, but consistent.  
An unbiased estimate

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^n \hat{e}_i^2}{n-2}$$

Note:  $\sum \hat{e}_i^2$  is the SSE

## Sampling Experiment

Consider 10 samples of size 40

$$n = 40$$

$$y_1 = \beta_1 + \beta_2 x + e_1$$

$$y_2 = \beta_1 + \beta_2 x + e_2$$

:

$$y_{10} = \beta_1 + \beta_2 x + e_{10}$$

Ten sets of estimates. Variation  
in These = Sampling Variation.

Table 2-2

list ylist =  $y_1 \ y_2 \ \dots \ y_{10}$

loop foreach i in ylist -- progressive

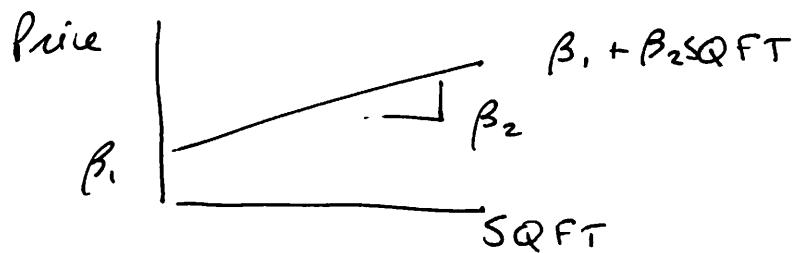
OLS ylist.\$i o x

end loop.

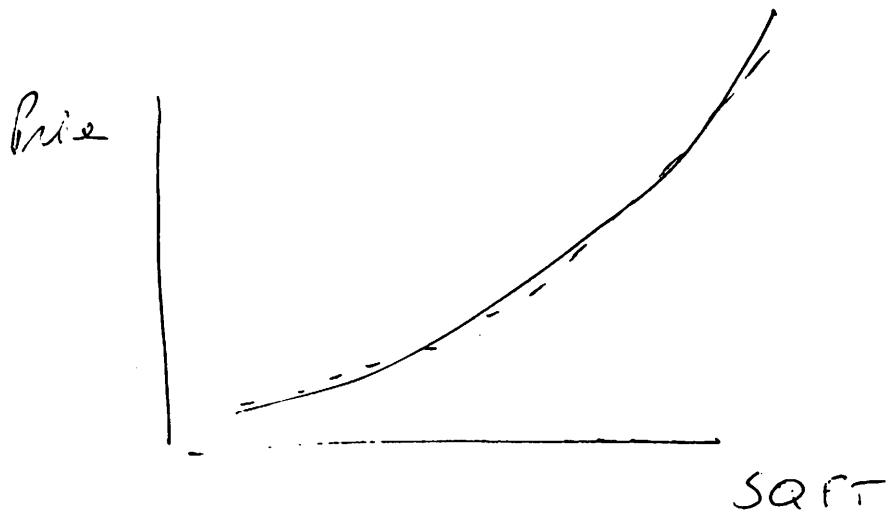
Generating Random Sample.

## Nonlinear Relationships

$$\text{Price} = \beta_1 + \beta_2 \text{SQFT} + e.$$



$$\text{Price} = \alpha_1 + \alpha_2 \text{SQFT}^2 + e.$$



$$\frac{dP}{dS} = 2\alpha_2 \text{SQFT}$$

depends on SQFT.

if  $\alpha_2 > 0$  Then



## Regression with Indicators

Let  $UTOWN = \begin{cases} 1 & \text{if house is in} \\ & UTOWN} \\ 0 & \text{else.} \end{cases}$

$$\text{Price} = \beta_1 + \beta_2 UTOWN + e$$

$$E(\text{Price} | UTOWN = 0) = \beta_1$$

$$E(\text{Price} | UTOWN = 1) = \beta_1 + \beta_2$$

