Testing Joint hypotheses

- 2 or more hypotheses to test -
~~Both~~ Assumed to Be all True under H_0:
At least 1 of the hypotheses ~~with~~ is false
then You Reject H_0:

Based on an F-Test.

Recall χ^2_n & χ^2_m independent R.V.

$$F = \frac{\chi^2_m / m}{\chi^2_n / n} \sim F_{m,n}$$

so, F-Tests are Basically Ratio's, just like t-test - Except you are taking Ratio of 2 Chi-squares.

Basic Idea

Hypotheses represent possible restrictions that can be imposed on a model.

$$tr = \beta_1 + \beta_2 A_t + \beta_3 P_t + e_t$$

$$H_0: \beta_2 = 0$$
$$H_A: \beta_2 \neq 0$$

The null says A has no effect. Imposing this

$$tr = \beta_1 + 0 \cdot A_t + \beta_3 P_t + e_t$$
$$= \beta_1 + \beta_3 P_t + e_t$$

OR

suppose A_t offsets P_t in the sense

$$H_0: \beta_2 + \beta_3 = 0$$
$$H_A: \text{not } H_0$$

Restriction $\Rightarrow$ $\beta_2 = -\beta_3$

$$tr = \beta_1 + (-\beta_3 A_t) + \beta_3 P_t + e_t$$
$$= \beta_1 + \beta_3 (P_t - A_t) + e_t$$

create new variable

$$X_t^* = P_t - A_t$$

use OLS

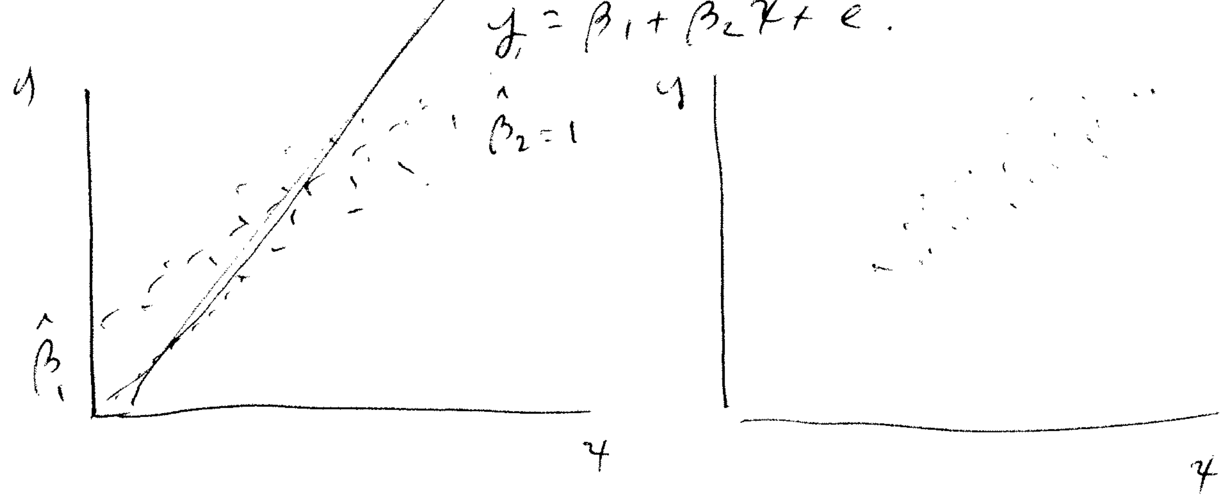
$$tr = \beta_1 + \beta_3 X_1^* + e_t$$

to get $\hat{\beta}_2 = -\hat{\beta}_3$

Using LS on Restricted Model is called Restricted LS.

Test

The test of the Restriction compares SSE from Rest and Unrest Models that are estimated By LS.



Suppose I restrict $\beta_1 = 0$

LS fit won't be as good when I impose Restriction

$$SSE_R \geq SSE_u$$

The worse the Restriction, the Bigger the SSE_R will get.

If SSE_R is enough larger than SSE_u then we have evidence that Rest. is NOT True.

Test Stat

$$F = \frac{(SSE_R - SSE_u)/J}{SSE_u/T-K}$$

SSE_R - Sum of squared errors from rest model

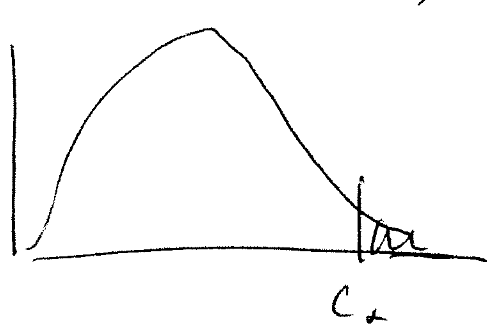
SSE_u " " " from unrest.

J # of hypotheses.

T sample size

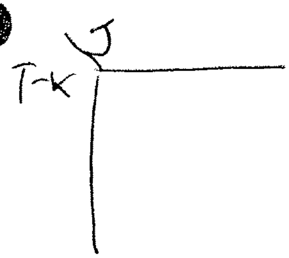
K # of params in unrest model.

If H_0 true, $F \sim F_{J,T-K}$. Choose α and find c_α



If $F > c_\alpha$ reject.

Table 3 for crit Values



Example

Overall - F Test

Test for overall Regression Significance

$$y_t = \beta_1 + \beta_2 x_{t2} + \cdots + \beta_K x_{tK} + e_t$$

If all slopes are zero $\Rightarrow$ no Model

$$y_t = \beta_1 + e_t$$

$H_0: \beta_2 = \beta_3 = \cdots = \beta_K = 0$

H_A: At least 1 $\beta_i \neq 0$ $i = 2, \ldots, K$

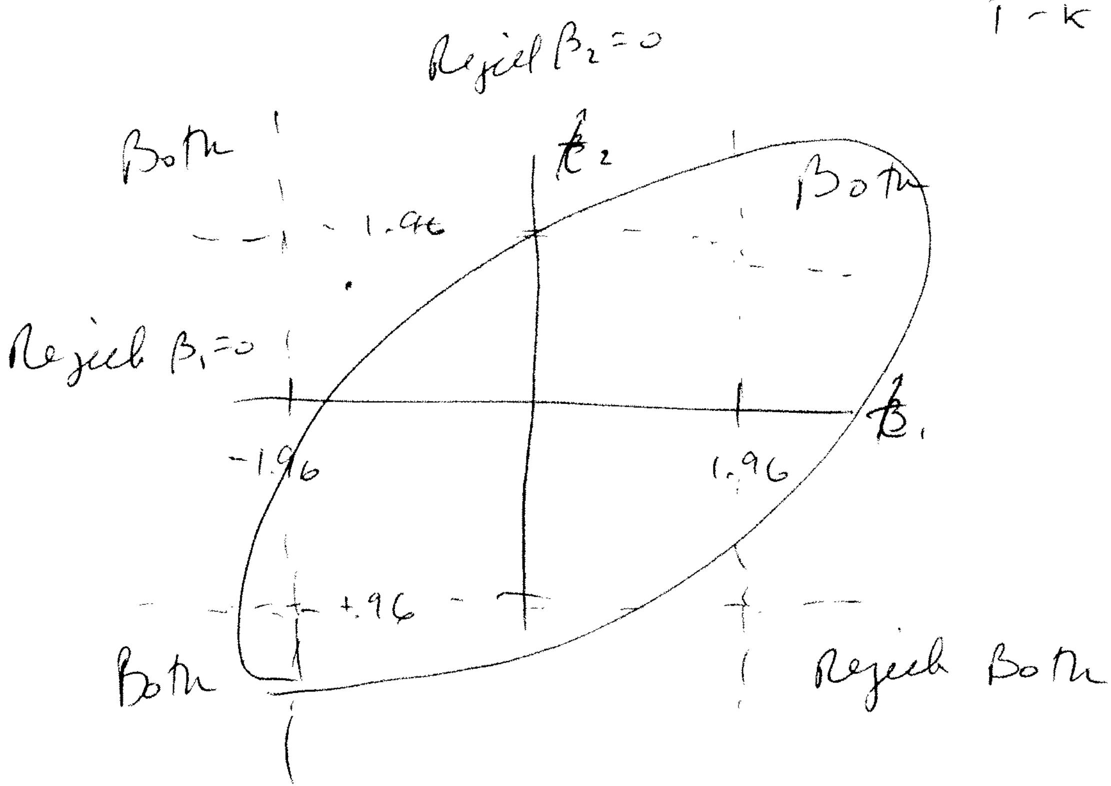
This amounts to $K-1$ restrictions

Note: J = # params in unrest − # params in rest.

$= K - 1$

Not the same as individual t-tests

$t_{T-K>120}$



F stat is ellipse
and takes into account
that $Cov(\hat{\beta}_1, \hat{\beta}_2) \neq 0$

So, you would not reject $\beta_1, \beta_2 = 0$
By t-stats But be in
Reject Region of Joint test.

Or in Not reject Region of F
But Reject Both in t-tests.

Gretl

Restrict

b1 = 0

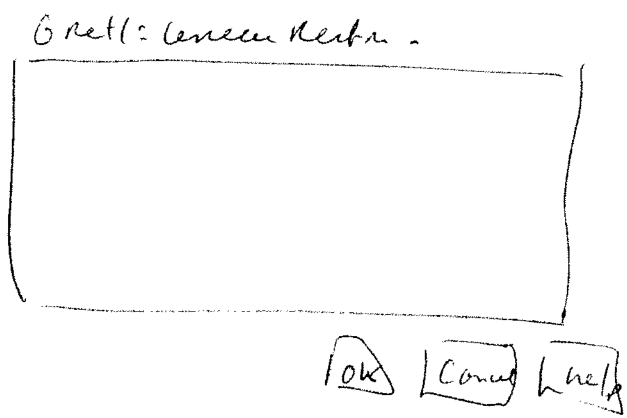
b1 + 2*b3 = 1

b4 - b5 = 0

end restrict

Estimate Model

Tests > Linear Restrictions



Restriction set

1: b(CPI) = 0
2: b(PoP) + 2*b(packPc) = 1
3: b(

Test Stat $F(3, 91) = 1625.5$

P val = 6.95 e-079

$< \alpha$
Reject