

## Hypothesis testing

Many economic decision problems require some means of determining whether or not a parameter is a specific value or whether its sign is positive or negative.

Hypothesis testing is a procedure that allows us to compare a conjecture about pop'l. parameters to information contained in a sample.

Every test has four elements

1. null hypothesis
2. Alternative hypothesis
3. test statistic
4. Rejection region.

### Null hypothesis $H_0$ :

specifies a value for the parameter in question. A null hypothesis is the belief that will be held until we ~~can~~ are convinced by sample evidence that it is untrue. In this case, we reject  $H_0$ .

## Alternative Hypothesis

This is the fall back position taken if  $H_0$  rejected. The alternative is important and often determines how powerful your test is and the nature of your rejection region.

$$Y_t = \beta_0 + \beta_1 X_t + \epsilon_t$$

$$H_0: \beta_1 = 0$$

There are three possibilities for  $H_A$ :

- $\beta_1 \neq 0$  either  $+ \alpha - \text{but}$   
not zero.
- $\beta_1 > 0$  rejection of  $\beta_1 = 0$   
 $\Rightarrow \beta_1 > 0$ . we  
completely disown  
the poss. that  $\beta_1 < 0$   
logically unacceptable
- $\beta_1 < 0$  rejection of  $\beta_1 = 0$   
 $\Rightarrow \beta_1 < 0$  and  
 $\beta_1 > 0$  logically  
unacceptable.

The last 2 alternative contain more information. More info usually means more power, more efficient use of data.

### Test stat

The sample info about  $H_0$  is embedded into the test stat.

The test stat is a random variable with a known p.d.f. when  $H_0$  is true.

$$H_0: \beta_2 = 0$$

$$H_A: \beta_2 \neq 0$$

$$\frac{b_2 - \beta_2}{\sqrt{\text{Var}(b_2)}} \sim t_{r-2}$$

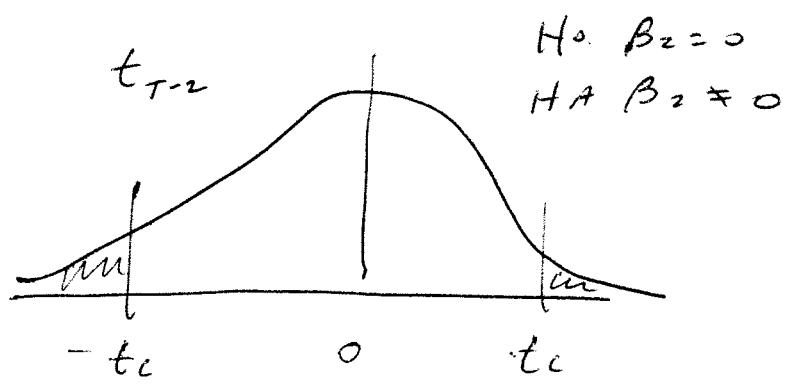
under  $H_0: \beta_2 = 0$

$$\frac{b_2}{\sqrt{\text{Var}(b_2)}} \sim t_{r-2}$$

As  $b_2$ , our estimate of  $\beta_2$ , differs from 0 this r.v. will get bigger. Evidence that  $\beta_2 \neq 0$ .

## Rejection Region

This amounts to specifying the values of the test stat that lead to rejection of  $H_0$ ; It is a set of values that the test stat takes on with low prob when  $H_0$  is true.



If  $\beta_2$  is close to zero, then it is more likely  $\beta_2 = 0$

$$t = \frac{b_2}{\sqrt{\text{Var}(b_2)}} \sim t_{T-2} \quad \text{or}$$

Only when  $b_2$ :  $t < -t_c$  or  $t > t_c$  does it become likely that  $\beta_2 \neq 0$   
So, Picking the desired Probability

of rejecting when  $H_0$  is true is called choosing the significance level of the test.

Finding  $t_c$  s.t. that

$$P(t > t_c) = P(t < -t_c) = \alpha/2$$

defines the rejection region for the test.

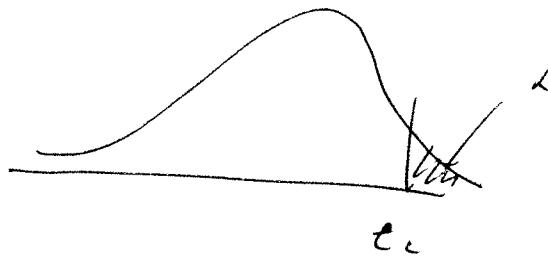
or.  $P(-t_c < t < t_c) = 1 - \alpha$

reject if  $|t| \geq t_c$

One side

$$H_0: \beta_2 = 0$$

$$H_A: \beta_2 > 0$$



Since  $t$  is unlikely to be negative and only expected to be 0 or  $> 0$  then the rejection region is in the right (pos) tail of the  $t$ -dist.

if  $t > t_c$  Reject.

Choose  $t_c$  so that  $\alpha$  is the tail prob-

## Format

1. Determine  $H_0$  and  $H_a$
2. Specify test stat and its dist when  $H_0$  true.
3. Select  $\alpha$
4. Compute test stat
5. Compare stat to crit value and state decision.

## Type I & Type II errors

There are 2 kinds of mistakes one can make when making a decision based on a hypothesis test.

Type I

- $H_0$  is true, but we reject it
- $H_0$  is false, but we fail to reject it

Type II.

The prob of type I error is the signif level of your test,  $\alpha$ . This is the kind of error that is under your control.

Type II error depends on the value of the unknown parameters. ( $H_0$  is true) These are unknowable and hence the level of type II error is not controllable.

- Notes:
- \* Prob of type I and type II error are inversely related.  
Choosing a very small increases the prob of not rejecting a false null.
  - \* closer the hypothesized parameter value is to its actual value, the larger the prob of type II err. The test has power to discriminate between true parameter values and the false hypothesized value if the two are very close.
  - \* larger  $T$ , smaller prob of type II err. given  $\alpha$ .

## P-Value

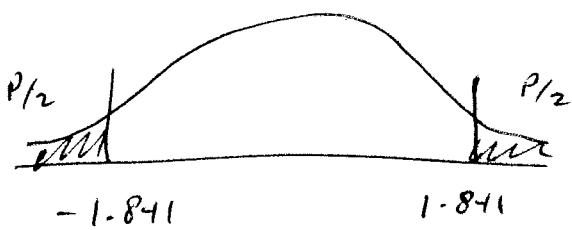
When reporting the outcome of statistical tests it has become popular to compute p-value associated with the computed value of the statistic.

Basically, the P-value measures the tail area of the distribution based on the computed value of your statistic.

$$H_0: \beta_1 = 0 \quad H_A: \beta_1 \neq 0$$

$$t = \frac{\hat{\beta}_1 - 0}{\sqrt{\text{Var}(\hat{\beta}_1)}} \sim t_{n-2} \text{ if } H_0 \text{ true.}$$

$$t = \frac{40.767}{22.138} = 1.841$$



$$\text{computed } P/2 + P/2 = .0734 > .05$$

∴ would not reject at  $\alpha = .05$  level.

If P-val is smaller than  $\alpha$ , Reject  
If not, do not reject.