

Small Sample Performance of Instrumental Variables Probit Estimators: A Monte Carlo Investigation

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Motivation Estimators

LIML
Newey
Small Sample Performance?

Design

Goals
Equations
Regressors and Errors
Parameters

Results

Example
Reduced Form
Some Things Change
Others Don't
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Motivation

- ▶ Does managerial compensation affect the decision to hedge using foreign exchange derivatives?

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- ▶ Some of the compensation variables are endogenous.
- ▶ Consistent estimation and hypothesis testing using Instrumental Variables.
- ▶ Stata offers 2 choices.

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1. Two-step with Murphy-Topel covariance
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- ▶ Asymptotically normally distributed
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- ▶ Approximate significance tests of parameters are statistically valid and, if the MLE can be computed, the tests are easy to compute

Newey's (two-step) estimator–AGLS

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This estimator will almost certainly be computable.

- ▶ Asymptotically normally distributed
- ▶ Asymptotically efficient in some cases
- ▶ Approximate significance tests of parameters are statistically valid and easy to compute
- ▶ Much easier to compute the estimators, making it possible to bootstrap or jackknife

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- ▶ ML

Design Goals

The basic design was first used by Rivers and Vuong. They vary degree of correlation between probit and the reduced form to study the bias and mse of several estimators.

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- ▶ Instrument Strength – RV consider only very strong instruments in their design.
- ▶ Different proportions of 1s and 0s are considered (no effect)
- ▶ Minimize the scaling problem
- ▶ Focus on significance test rather than bias

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- ▶ (Probit) The underlying regression equation:

$$y_{1i}^* = \gamma y_{2i} + \beta_1 + \beta_2 x_{2i} + u_i \quad (1)$$

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- ▶ (Reduced Form) In the just identified case, the endogenous regressor y_{2i} is determined

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- ▶ and the over-identified case,

$$y_{2i} = \pi_1 + \pi_2 x_{2i} + \pi_3 x_{3i} + \pi_4 x_{4i} + \nu_i \quad (3)$$

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- ▶ ν_i and η_i standard normals
- ▶ λ is varied on the interval $[-2, 2]$ to generate correlation between the endogenous explanatory variable and the regression's error.

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- ▶ Reduced Form: $\theta\pi$ where

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- ▶ Sample sizes: 200 and 1000

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- ▶ When there is no endogeneity, ols and probit work well (as expected).
- ▶ It is clear that OLS and Probit should be avoided when you have an endogenous regressor.
- ▶ Linear instrumental variables can be used for significance testing, though their performance is not as good as AGLS. The Linear IV estimator performs better when the model is just identified.

Weak Instruments and Size

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- ▶ Size of IVP is acceptable. Puzzling and deserves more study.
- ▶ The size of the significance tests based on the AGLS estimator is reasonable, but the standard errors are too small—a situation that gets worse as severity of the endogeneity problem increases. When instruments are very weak, the actual test size can be double the nominal.

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- ▶ Pretesting for endogeneity doesn't help. When Instruments are extremely weak it is outperformed by the other estimators considered, except when the no endogeneity hypothesis is true (and probit should be used).
- ▶ ML tests are better if the sample is large (1000) or instruments strong. In small samples with weak instruments, AGLS is better for significance testing (size).

Summary Results from Reduced-form Equations

| Instruments | Reduced Form Equation | | |
|----------------------------|-----------------------|----------|-------|
| | Leverage | Options | Bonus |
| | Coefficient | P-values | |
| Number of Employees | 0.182 | 0.000 | 0.000 |
| Number of Subsidiaries | 0.000 | 0.164 | 0.008 |
| Number of Offices | 0.248 | 0.000 | 0.000 |
| CEO Age | 0.026 | 0.764 | 0.572 |
| 12 Month Maturity Mismatch | 0.353 | 0.280 | 0.575 |
| CFA | 0.000 | 0.826 | 0.368 |
| R-Square | 0.296 | 0.698 | 0.606 |

Parameters that change significance

| | AGLS | ML |
|----------------------|----------------------|----------------------|
| Leverage | 21.775 (0.104) | 12.490 (0.021) |
| Total Assets | 0.365 (0.032) | 0.190 (0.183) |
| Return on Equity | -0.034 (0.230) | -0.020 (0.083) |
| Market-to-Book ratio | -0.002 (0.132) | -0.001 (0.098) |
| Dividends Paid | -8.43E-07 (0.134) | -4.84E-07 (0.044) |

Parameters that are significant in both

- ▶ Option Awards
- ▶ Bonuses
- ▶ Insider Ownership
- ▶ Institutional Ownership

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<http://www.LearnEconometrics.com/pdf/JSM2008.pdf>

Thanks!

Table 1a Bias of each estimator based on samples of size 200. Monte Carlo used 1000 samples.
The model is just identified. The approximate proportion of 1's in each sample is .5.

| Design | | Estimator | | | | | | |
|----------|-----------|-----------|--------|-----------|-----------|--------|--------|---------|
| θ | λ | ols | probit | IV probit | Linear IV | agls | tscml | pretest |
| 0.05 | 2 | 0.818 | 2.103 | -6.807 | -1.533 | -1.858 | -1.858 | 0.699 |
| 0.05 | 1 | 0.575 | 1.034 | 2.934 | 1.005 | 1.572 | 1.572 | 1.082 |
| 0.05 | 0.5 | 0.326 | 0.510 | -6.885 | -3.057 | -3.717 | -3.717 | -0.600 |
| 0.05 | 0 | 0.004 | 0.006 | -12.681 | -7.284 | -8.732 | -8.732 | 0.105 |
| 0.05 | -0.5 | -0.330 | -0.515 | -5.085 | -2.915 | -4.721 | -4.721 | -0.210 |
| 0.05 | -1 | -0.573 | -1.028 | -0.853 | -0.834 | -0.302 | -0.302 | -0.700 |
| 0.05 | -2 | -0.817 | -2.078 | -1.478 | -0.972 | -2.429 | -2.429 | -1.980 |
| 0.1 | 2 | 0.813 | 2.043 | 22.393 | 6.184 | 7.702 | 7.702 | 8.046 |
| 0.1 | 1 | 0.572 | 1.023 | 3.000 | 0.041 | -0.423 | -0.423 | 0.446 |
| 0.1 | 0.5 | 0.324 | 0.509 | 1.580 | 0.473 | 0.960 | 0.960 | 0.628 |
| 0.1 | 0 | -0.001 | -0.001 | 12.316 | 6.766 | 8.767 | 8.767 | 0.007 |
| 0.1 | -0.5 | -0.328 | -0.510 | -0.196 | -0.182 | -0.405 | -0.405 | -0.324 |
| 0.1 | -1 | -0.570 | -1.020 | 0.251 | 0.095 | 0.221 | 0.221 | -0.217 |
| 0.1 | -2 | -0.813 | -2.037 | -0.069 | -0.052 | -0.285 | -0.285 | -1.023 |
| 0.25 | 2 | 0.785 | 1.848 | -0.625 | -0.188 | -0.508 | -0.508 | -0.482 |
| 0.25 | 1 | 0.547 | 0.966 | -0.286 | -0.137 | -0.199 | -0.199 | -0.010 |
| 0.25 | 0.5 | 0.312 | 0.488 | -0.127 | -0.104 | -0.075 | -0.075 | 0.189 |
| 0.25 | 0 | -0.005 | -0.004 | 0.027 | -0.057 | 0.018 | 0.018 | -0.016 |
| 0.25 | -0.5 | -0.317 | -0.487 | 0.150 | 0.040 | 0.143 | 0.143 | -0.111 |
| 0.25 | -1 | -0.550 | -0.965 | 0.183 | 0.111 | 0.273 | 0.273 | 0.049 |
| 0.25 | -2 | -0.782 | -1.840 | 0.288 | 0.175 | 0.456 | 0.456 | 0.400 |
| 0.5 | 2 | 0.694 | 1.390 | -0.086 | -0.030 | -0.053 | -0.053 | -0.053 |
| 0.5 | 1 | 0.485 | 0.809 | -0.065 | -0.039 | -0.040 | -0.040 | -0.031 |
| 0.5 | 0.5 | 0.274 | 0.425 | -0.045 | -0.041 | -0.029 | -0.029 | 0.055 |
| 0.5 | 0 | -0.005 | -0.002 | -0.005 | -0.031 | -0.004 | -0.004 | -0.006 |
| 0.5 | -0.5 | -0.283 | -0.427 | 0.014 | -0.014 | 0.013 | 0.013 | -0.070 |
| 0.5 | -1 | -0.487 | -0.807 | 0.036 | 0.015 | 0.049 | 0.049 | 0.040 |
| 0.5 | -2 | -0.696 | -1.385 | 0.030 | 0.013 | 0.056 | 0.056 | 0.056 |
| 1 | 2 | 0.478 | 0.738 | 0.005 | -0.001 | 0.004 | 0.004 | 0.004 |
| 1 | 1 | 0.335 | 0.505 | -0.003 | -0.008 | -0.002 | -0.002 | -0.002 |
| 1 | 0.5 | 0.186 | 0.280 | 0.001 | -0.011 | 0.001 | 0.001 | 0.010 |
| 1 | 0 | -0.004 | 0.002 | 0.009 | -0.010 | 0.006 | 0.006 | 0.004 |
| 1 | -0.5 | -0.198 | -0.285 | 0.007 | -0.006 | 0.007 | 0.007 | -0.001 |
| 1 | -1 | -0.338 | -0.498 | 0.011 | 0.001 | 0.016 | 0.016 | 0.016 |
| 1 | -2 | -0.480 | -0.730 | 0.014 | 0.006 | 0.028 | 0.028 | 0.028 |

Table 1b Bias of each estimator based on samples of size 1000. Monte Carlo used 1000 samples.
The model is just identified. The approximate proportion of 1's in each sample is .5.

| Design | | Estimator | | | | | | |
|----------|-----------|-----------|--------|-----------|-----------|---------|---------|---------|
| θ | λ | ols | probit | IV probit | Linear IV | agsl | tscml | pretest |
| 0.05 | 2 | 0.811 | 2.008 | 1.397 | 0.382 | 0.551 | 0.551 | 0.551 |
| | 1 | 0.572 | 1.008 | 0.474 | 0.089 | 0.212 | 0.212 | 0.212 |
| | 0.5 | 0.327 | 0.501 | -0.158 | -0.056 | -0.310 | -0.310 | -0.310 |
| | 0 | 0.000 | 0.000 | 1.266 | 0.204 | 0.895 | 0.895 | 0.895 |
| | -0.5 | -0.328 | -0.501 | -1.216 | -0.770 | -1.386 | -1.386 | -1.386 |
| | -1 | -0.569 | -1.001 | -10.904 | -7.669 | -14.615 | -14.615 | -14.615 |
| | -2 | -0.811 | -2.011 | -1.135 | -0.761 | -1.850 | -1.850 | -1.850 |
| 0.1 | 2 | 0.808 | 1.982 | -0.229 | -0.087 | -0.196 | -0.196 | -0.196 |
| | 1 | 0.568 | 0.997 | -3.672 | -1.381 | -1.869 | -1.869 | -1.869 |
| | 0.5 | 0.326 | 0.499 | -0.923 | -0.448 | -0.549 | -0.549 | -0.549 |
| | 0 | -0.002 | -0.002 | -0.092 | -0.112 | -0.065 | -0.065 | -0.065 |
| | -0.5 | -0.328 | -0.501 | -0.072 | -0.075 | -0.095 | -0.095 | -0.095 |
| | -1 | -0.567 | -0.993 | 0.136 | 0.072 | 0.184 | 0.184 | 0.184 |
| | -2 | -0.809 | -1.981 | -0.208 | -0.137 | -0.227 | -0.227 | -0.227 |
| 0.25 | 2 | 0.778 | 1.782 | -0.040 | -0.017 | -0.029 | -0.029 | -0.029 |
| | 1 | 0.547 | 0.946 | -0.023 | -0.022 | -0.017 | -0.017 | -0.017 |
| | 0.5 | 0.314 | 0.481 | -0.026 | -0.030 | -0.016 | -0.016 | -0.016 |
| | 0 | -0.002 | -0.001 | 0.001 | -0.021 | 0.001 | 0.001 | 0.001 |
| | -0.5 | -0.316 | -0.481 | 0.023 | -0.004 | 0.023 | 0.023 | 0.023 |
| | -1 | -0.547 | -0.944 | 0.015 | -0.001 | 0.021 | 0.021 | 0.021 |
| | -2 | -0.779 | -1.779 | 0.039 | 0.019 | 0.058 | 0.058 | 0.058 |
| 0.5 | 2 | 0.690 | 1.352 | 0.003 | -0.002 | 0.002 | 0.002 | 0.002 |
| | 1 | 0.484 | 0.795 | -0.002 | -0.007 | 0.000 | 0.000 | 0.000 |
| | 0.5 | 0.278 | 0.418 | -0.001 | -0.010 | -0.001 | -0.001 | -0.001 |
| | 0 | -0.002 | 0.000 | -0.003 | -0.012 | -0.002 | -0.002 | -0.002 |
| | -0.5 | -0.279 | -0.417 | 0.005 | -0.005 | 0.005 | 0.005 | 0.005 |
| | -1 | -0.486 | -0.796 | -0.003 | -0.009 | -0.003 | -0.003 | -0.003 |
| | -2 | -0.689 | -1.344 | 0.010 | 0.004 | 0.014 | 0.014 | 0.014 |
| 1 | 2 | 0.474 | 0.719 | -0.002 | -0.002 | -0.004 | -0.004 | -0.004 |
| | 1 | 0.331 | 0.491 | -0.002 | -0.004 | 0.000 | 0.000 | 0.000 |
| | 0.5 | 0.190 | 0.279 | -0.002 | -0.005 | -0.001 | -0.001 | -0.001 |
| | 0 | -0.001 | 0.002 | 0.004 | -0.004 | 0.003 | 0.003 | 0.003 |
| | -0.5 | -0.193 | -0.277 | 0.000 | -0.005 | 0.000 | 0.000 | 0.000 |
| | -1 | -0.334 | -0.492 | 0.002 | -0.002 | 0.003 | 0.003 | 0.003 |
| | -2 | -0.475 | -0.721 | 0.000 | -0.002 | -0.001 | -0.001 | -0.001 |

Table 1c Bias of each estimator based on samples of size 200. Monte Carlo used 1000 samples.
The model is overidentified. The approximate proportion of 1's in each sample is .5.

| θ | λ | Design | | | | | | Estimator |
|----------|-----------|--------|--------|-----------|-----------|--------|--------|-----------|
| | | ols | probit | IV probit | Linear IV | agsl | tscml | |
| 0.050 | 2.000 | 0.830 | 2.078 | 2.376 | 0.668 | 1.707 | 1.692 | 1.789 |
| 0.050 | 1.000 | 0.592 | 1.030 | 0.989 | 0.302 | 0.642 | 0.650 | 0.803 |
| 0.050 | 0.500 | 0.342 | 0.515 | 0.613 | 0.222 | 0.353 | 0.352 | 0.388 |
| 0.050 | 0.000 | -0.002 | -0.003 | 0.039 | -0.023 | 0.027 | 0.029 | -0.008 |
| 0.050 | -0.500 | -0.342 | -0.511 | -0.428 | -0.322 | -0.431 | -0.434 | -0.484 |
| 0.050 | -1.000 | -0.591 | -1.033 | -0.525 | -0.427 | -0.776 | -0.767 | -0.787 |
| 0.050 | -2.000 | -0.828 | -2.072 | -0.996 | -0.649 | -1.701 | -1.694 | -1.931 |
| 0.100 | 2.000 | 0.823 | 2.047 | 1.227 | 0.333 | 0.946 | 0.938 | 1.164 |
| 0.100 | 1.000 | 0.587 | 1.018 | 0.598 | 0.176 | 0.374 | 0.374 | 0.564 |
| 0.100 | 0.500 | 0.339 | 0.508 | 0.287 | 0.069 | 0.163 | 0.163 | 0.316 |
| 0.100 | 0.000 | 0.000 | 0.001 | -0.015 | -0.073 | -0.010 | -0.011 | -0.034 |
| 0.100 | -0.500 | -0.340 | -0.504 | -0.167 | -0.161 | -0.155 | -0.156 | -0.376 |
| 0.100 | -1.000 | -0.587 | -1.016 | -0.255 | -0.222 | -0.396 | -0.395 | -0.683 |
| 0.100 | -2.000 | -0.823 | -2.034 | -0.456 | -0.315 | -0.755 | -0.740 | -0.951 |
| 0.250 | 2.000 | 0.781 | 1.762 | 0.007 | -0.007 | -0.006 | -0.008 | 0.003 |
| 0.250 | 1.000 | 0.557 | 0.951 | 0.008 | -0.018 | 0.007 | 0.007 | 0.128 |
| 0.250 | 0.500 | 0.321 | 0.480 | 0.009 | -0.030 | 0.003 | 0.004 | 0.173 |
| 0.250 | 0.000 | -0.003 | 0.000 | 0.010 | -0.036 | 0.006 | 0.007 | -0.004 |
| 0.250 | -0.500 | -0.325 | -0.482 | -0.008 | -0.038 | -0.010 | -0.010 | -0.190 |
| 0.250 | -1.000 | -0.559 | -0.944 | 0.005 | -0.020 | 0.008 | 0.009 | -0.120 |
| 0.250 | -2.000 | -0.780 | -1.768 | 0.038 | 0.015 | 0.039 | 0.041 | 0.032 |
| 0.500 | 2.000 | 0.666 | 1.240 | 0.000 | -0.004 | -0.002 | -0.004 | -0.004 |
| 0.500 | 1.000 | 0.471 | 0.752 | -0.003 | -0.013 | -0.003 | -0.003 | 0.000 |
| 0.500 | 0.500 | 0.269 | 0.400 | -0.005 | -0.019 | -0.005 | -0.004 | 0.056 |
| 0.500 | 0.000 | -0.005 | 0.000 | -0.004 | -0.022 | -0.004 | -0.003 | 0.002 |
| 0.500 | -0.500 | -0.281 | -0.410 | -0.007 | -0.023 | -0.010 | -0.009 | -0.072 |
| 0.500 | -1.000 | -0.478 | -0.759 | 0.010 | -0.004 | 0.017 | 0.017 | 0.014 |
| 0.500 | -2.000 | -0.664 | -1.239 | 0.010 | 0.001 | 0.009 | 0.009 | 0.009 |
| 1.000 | 2.000 | 0.414 | 0.592 | 0.002 | -0.002 | -0.001 | -0.001 | -0.001 |
| 1.000 | 1.000 | 0.293 | 0.421 | 0.000 | -0.006 | -0.002 | -0.002 | -0.002 |
| 1.000 | 0.500 | 0.168 | 0.245 | -0.001 | -0.009 | -0.001 | -0.001 | 0.003 |
| 1.000 | 0.000 | -0.006 | -0.002 | -0.002 | -0.011 | -0.002 | -0.002 | -0.002 |
| 1.000 | -0.500 | -0.177 | -0.246 | 0.001 | -0.008 | 0.001 | 0.001 | -0.003 |
| 1.000 | -1.000 | -0.301 | -0.431 | -0.007 | -0.011 | -0.011 | -0.011 | -0.011 |
| 1.000 | -2.000 | -0.417 | -0.601 | 0.000 | -0.002 | 0.003 | 0.003 | 0.003 |

Table 1d Bias of each estimator based on samples of size 1000. Monte Carlo used 1000 samples.
The model is overidentified. The approximate proportion of 1's in each sample is .5.

| Design | | Estimator | | | | | | |
|----------|-----------|-----------|--------|-----------|-----------|--------|--------|---------|
| θ | λ | ols | probit | IV probit | Linear IV | agls | tscml | pretest |
| 0.05 | 2 | 0.817 | 2.007 | 0.873 | 0.276 | 0.649 | 0.650 | 0.953 |
| | 1 | 0.578 | 1.005 | 0.415 | 0.220 | 0.274 | 0.275 | 0.515 |
| | 0.5 | 0.333 | 0.500 | 0.214 | 0.172 | 0.116 | 0.117 | 0.327 |
| | 0 | 0.000 | 0.000 | -0.077 | 0.073 | -0.054 | -0.054 | 0.005 |
| | -0.5 | -0.333 | -0.502 | -0.086 | 0.044 | -0.088 | -0.088 | -0.255 |
| | -1 | -0.578 | -1.003 | -0.282 | -0.171 | -0.400 | -0.401 | -0.684 |
| | -2 | -0.815 | -2.002 | -0.413 | -0.243 | -0.694 | -0.695 | -0.930 |
| 0.1 | 2 | 0.811 | 1.966 | 0.270 | 0.094 | 0.171 | 0.171 | 0.208 |
| | 1 | 0.574 | 0.994 | 0.028 | 0.059 | 0.009 | 0.010 | 0.211 |
| | 0.5 | 0.332 | 0.499 | -0.019 | 0.062 | -0.007 | -0.007 | 0.216 |
| | 0 | 0.001 | -0.001 | -0.006 | 0.080 | -0.004 | -0.004 | -0.007 |
| | -0.5 | -0.329 | -0.496 | 0.016 | 0.079 | 0.023 | 0.023 | -0.198 |
| | -1 | -0.572 | -0.990 | -0.001 | 0.045 | 0.006 | 0.005 | -0.171 |
| | -2 | -0.811 | -1.968 | 0.041 | 0.044 | 0.075 | 0.074 | 0.040 |
| 0.25 | 2 | 0.775 | 1.739 | 0.008 | 0.009 | 0.009 | 0.010 | 0.010 |
| | 1 | 0.548 | 0.927 | -0.033 | 0.007 | -0.018 | -0.018 | -0.017 |
| | 0.5 | 0.319 | 0.476 | -0.008 | 0.025 | -0.005 | -0.005 | 0.035 |
| | 0 | 0.000 | -0.002 | 0.000 | 0.034 | 0.000 | 0.000 | 0.001 |
| | -0.5 | -0.315 | -0.473 | -0.001 | 0.027 | -0.001 | -0.001 | -0.044 |
| | -1 | -0.546 | -0.928 | -0.001 | 0.018 | -0.001 | -0.001 | -0.001 |
| | -2 | -0.774 | -1.730 | 0.002 | 0.008 | 0.002 | 0.002 | 0.002 |
| 0.5 | 2 | 0.667 | 1.248 | 0.015 | 0.008 | 0.011 | 0.011 | 0.011 |
| | 1 | 0.473 | 0.753 | 0.000 | 0.009 | -0.001 | -0.001 | -0.001 |
| | 0.5 | 0.274 | 0.399 | 0.000 | 0.014 | 0.001 | 0.001 | 0.001 |
| | 0 | 0.003 | -0.001 | 0.003 | 0.018 | 0.002 | 0.002 | -0.001 |
| | -0.5 | -0.269 | -0.398 | 0.002 | 0.015 | 0.002 | 0.002 | 0.002 |
| | -1 | -0.469 | -0.752 | -0.002 | 0.007 | -0.004 | -0.004 | -0.004 |
| | -2 | -0.667 | -1.243 | 0.000 | 0.004 | 0.000 | 0.000 | 0.000 |
| 1 | 2 | 0.429 | 0.617 | -0.004 | 0.001 | -0.003 | -0.003 | -0.003 |
| | 1 | 0.305 | 0.433 | 0.002 | 0.005 | 0.002 | 0.002 | 0.002 |
| | 0.5 | 0.178 | 0.249 | 0.001 | 0.008 | 0.001 | 0.001 | 0.001 |
| | 0 | 0.003 | -0.001 | -0.004 | 0.006 | -0.003 | -0.003 | -0.001 |
| | -0.5 | -0.171 | -0.248 | 0.001 | 0.008 | 0.000 | 0.000 | 0.000 |
| | -1 | -0.300 | -0.432 | 0.001 | 0.006 | 0.002 | 0.002 | 0.002 |
| | -2 | -0.428 | -0.617 | -0.002 | 0.000 | -0.003 | -0.003 | -0.003 |

Table 2a The size of 10% nominal tests. Only Linear IV and agls use consistent standard errors. N=200, mc=1000, just identified.

| Design | | Estimator | | | | | |
|----------|-----------|-----------|--------|-----------|-----------|-------|-------|
| θ | λ | ols | probit | IV probit | Linear IV | agls | tscml |
| 0.05 | 2 | 1.000 | 1.000 | 0.099 | 0.130 | 0.141 | 0.379 |
| | 1 | 1.000 | 1.000 | 0.096 | 0.046 | 0.110 | 0.197 |
| | 0.5 | 0.996 | 0.998 | 0.097 | 0.011 | 0.086 | 0.124 |
| | 0 | 0.099 | 0.099 | 0.104 | 0.002 | 0.092 | 0.107 |
| | -0.5 | 0.998 | 0.997 | 0.092 | 0.025 | 0.086 | 0.123 |
| | -1 | 1.000 | 1.000 | 0.082 | 0.049 | 0.108 | 0.194 |
| | -2 | 1.000 | 1.000 | 0.096 | 0.115 | 0.121 | 0.365 |
| 0.1 | 2 | 1.000 | 1.000 | 0.089 | 0.108 | 0.114 | 0.339 |
| | 1 | 1.000 | 1.000 | 0.092 | 0.045 | 0.102 | 0.193 |
| | 0.5 | 0.999 | 0.999 | 0.103 | 0.032 | 0.105 | 0.137 |
| | 0 | 0.099 | 0.088 | 0.110 | 0.008 | 0.102 | 0.111 |
| | -0.5 | 0.997 | 0.998 | 0.087 | 0.022 | 0.090 | 0.114 |
| | -1 | 1.000 | 1.000 | 0.091 | 0.067 | 0.110 | 0.192 |
| | -2 | 1.000 | 1.000 | 0.108 | 0.111 | 0.124 | 0.355 |
| 0.25 | 2 | 1.000 | 1.000 | 0.112 | 0.084 | 0.139 | 0.343 |
| | 1 | 1.000 | 1.000 | 0.104 | 0.084 | 0.141 | 0.216 |
| | 0.5 | 0.999 | 0.999 | 0.091 | 0.049 | 0.090 | 0.118 |
| | 0 | 0.105 | 0.106 | 0.092 | 0.052 | 0.089 | 0.094 |
| | -0.5 | 0.999 | 0.999 | 0.089 | 0.060 | 0.098 | 0.125 |
| | -1 | 1.000 | 1.000 | 0.085 | 0.083 | 0.117 | 0.188 |
| | -2 | 1.000 | 1.000 | 0.088 | 0.105 | 0.127 | 0.369 |
| 0.5 | 2 | 1.000 | 1.000 | 0.085 | 0.085 | 0.114 | 0.348 |
| | 1 | 1.000 | 1.000 | 0.093 | 0.084 | 0.114 | 0.192 |
| | 0.5 | 0.994 | 0.995 | 0.115 | 0.097 | 0.127 | 0.156 |
| | 0 | 0.097 | 0.101 | 0.113 | 0.094 | 0.111 | 0.114 |
| | -0.5 | 0.998 | 0.995 | 0.090 | 0.106 | 0.099 | 0.116 |
| | -1 | 1.000 | 1.000 | 0.099 | 0.098 | 0.122 | 0.193 |
| | -2 | 1.000 | 1.000 | 0.086 | 0.105 | 0.129 | 0.386 |
| 1 | 2 | 1.000 | 1.000 | 0.086 | 0.102 | 0.139 | 0.370 |
| | 1 | 1.000 | 1.000 | 0.087 | 0.095 | 0.114 | 0.200 |
| | 0.5 | 0.953 | 0.957 | 0.091 | 0.094 | 0.102 | 0.123 |
| | 0 | 0.108 | 0.101 | 0.103 | 0.101 | 0.098 | 0.105 |
| | -0.5 | 0.976 | 0.966 | 0.095 | 0.111 | 0.104 | 0.132 |
| | -1 | 1.000 | 1.000 | 0.089 | 0.104 | 0.115 | 0.202 |
| | -2 | 1.000 | 1.000 | 0.073 | 0.092 | 0.112 | 0.379 |

Table 2b Compute rejection rate for 10% nominal t-tests. Standard errors for agls and Linear IV are consistent. N=1000, mc=1000, model is just identified.

| Design | | Estimator | | | | | |
|----------|-----------|-----------|--------|-----------|-----------|-------|-------|
| θ | λ | ols | probit | IV probit | Linear IV | agls | tscml |
| 0.05 | 2 | 1.000 | 1.000 | 0.106 | 0.102 | 0.116 | 0.364 |
| | 1 | 1.000 | 1.000 | 0.086 | 0.051 | 0.103 | 0.180 |
| | 0.5 | 1.000 | 1.000 | 0.097 | 0.024 | 0.108 | 0.132 |
| | 0 | 0.107 | 0.108 | 0.102 | 0.005 | 0.098 | 0.103 |
| | -0.5 | 1.000 | 1.000 | 0.100 | 0.036 | 0.107 | 0.134 |
| | -1 | 1.000 | 1.000 | 0.079 | 0.062 | 0.101 | 0.178 |
| | -2 | 1.000 | 1.000 | 0.085 | 0.110 | 0.124 | 0.348 |
| 0.1 | 2 | 1.000 | 1.000 | 0.090 | 0.090 | 0.121 | 0.359 |
| | 1 | 1.000 | 1.000 | 0.080 | 0.062 | 0.101 | 0.173 |
| | 0.5 | 1.000 | 1.000 | 0.091 | 0.044 | 0.096 | 0.115 |
| | 0 | 0.092 | 0.101 | 0.122 | 0.043 | 0.120 | 0.121 |
| | -0.5 | 1.000 | 1.000 | 0.105 | 0.057 | 0.104 | 0.131 |
| | -1 | 1.000 | 1.000 | 0.098 | 0.084 | 0.119 | 0.192 |
| | -2 | 1.000 | 1.000 | 0.089 | 0.088 | 0.129 | 0.345 |
| 0.25 | 2 | 1.000 | 1.000 | 0.082 | 0.086 | 0.122 | 0.339 |
| | 1 | 1.000 | 1.000 | 0.078 | 0.070 | 0.113 | 0.184 |
| | 0.5 | 1.000 | 1.000 | 0.103 | 0.076 | 0.118 | 0.137 |
| | 0 | 0.101 | 0.112 | 0.111 | 0.091 | 0.111 | 0.111 |
| | -0.5 | 1.000 | 1.000 | 0.095 | 0.089 | 0.112 | 0.130 |
| | -1 | 1.000 | 1.000 | 0.086 | 0.089 | 0.112 | 0.190 |
| | -2 | 1.000 | 1.000 | 0.080 | 0.077 | 0.116 | 0.327 |
| 0.5 | 2 | 1.000 | 1.000 | 0.077 | 0.086 | 0.130 | 0.343 |
| | 1 | 1.000 | 1.000 | 0.069 | 0.071 | 0.102 | 0.172 |
| | 0.5 | 1.000 | 1.000 | 0.110 | 0.091 | 0.121 | 0.139 |
| | 0 | 0.094 | 0.099 | 0.106 | 0.097 | 0.104 | 0.106 |
| | -0.5 | 1.000 | 1.000 | 0.092 | 0.092 | 0.096 | 0.116 |
| | -1 | 1.000 | 1.000 | 0.087 | 0.102 | 0.110 | 0.198 |
| | -2 | 1.000 | 1.000 | 0.089 | 0.089 | 0.118 | 0.351 |
| 1 | 2 | 1.000 | 1.000 | 0.087 | 0.096 | 0.131 | 0.351 |
| | 1 | 1.000 | 1.000 | 0.079 | 0.080 | 0.108 | 0.177 |
| | 0.5 | 1.000 | 1.000 | 0.089 | 0.093 | 0.107 | 0.124 |
| | 0 | 0.099 | 0.102 | 0.097 | 0.090 | 0.096 | 0.096 |
| | -0.5 | 1.000 | 1.000 | 0.098 | 0.092 | 0.107 | 0.134 |
| | -1 | 1.000 | 1.000 | 0.090 | 0.104 | 0.122 | 0.203 |
| | -2 | 1.000 | 1.000 | 0.093 | 0.110 | 0.141 | 0.382 |

Table 2c The size of 10% nominal tests. Only Linear IV and agls use consistent standard errors. N=200, mc=1000, model is overidentified.

| Design | | Estimator | | | | | |
|----------|-----------|-----------|--------|-----------|-----------|-------|-------|
| θ | λ | ols | probit | IV probit | Linear IV | agls | tscml |
| 0.050 | 2.000 | 1.000 | 1.000 | 0.143 | 0.235 | 0.198 | 0.460 |
| | 1.000 | 1.000 | 1.000 | 0.129 | 0.107 | 0.156 | 0.258 |
| | 0.500 | 1.000 | 1.000 | 0.123 | 0.047 | 0.137 | 0.163 |
| | 0.000 | 0.098 | 0.086 | 0.111 | 0.007 | 0.102 | 0.113 |
| | -0.500 | 1.000 | 0.999 | 0.122 | 0.052 | 0.125 | 0.159 |
| | -1.000 | 1.000 | 1.000 | 0.113 | 0.124 | 0.140 | 0.238 |
| | -2.000 | 1.000 | 1.000 | 0.137 | 0.232 | 0.195 | 0.442 |
| 0.100 | 2.000 | 1.000 | 1.000 | 0.134 | 0.238 | 0.198 | 0.451 |
| | 1.000 | 1.000 | 1.000 | 0.111 | 0.099 | 0.129 | 0.223 |
| | 0.500 | 0.999 | 0.998 | 0.100 | 0.046 | 0.099 | 0.122 |
| | 0.000 | 0.105 | 0.111 | 0.106 | 0.020 | 0.099 | 0.111 |
| | -0.500 | 0.997 | 0.997 | 0.096 | 0.063 | 0.099 | 0.117 |
| | -1.000 | 1.000 | 1.000 | 0.095 | 0.118 | 0.124 | 0.204 |
| | -2.000 | 1.000 | 1.000 | 0.111 | 0.209 | 0.156 | 0.395 |
| 0.250 | 2.000 | 1.000 | 1.000 | 0.087 | 0.118 | 0.128 | 0.370 |
| | 1.000 | 1.000 | 1.000 | 0.115 | 0.121 | 0.132 | 0.221 |
| | 0.500 | 1.000 | 0.999 | 0.103 | 0.085 | 0.108 | 0.133 |
| | 0.000 | 0.108 | 0.115 | 0.113 | 0.076 | 0.110 | 0.115 |
| | -0.500 | 0.999 | 0.999 | 0.090 | 0.096 | 0.100 | 0.127 |
| | -1.000 | 1.000 | 1.000 | 0.088 | 0.123 | 0.112 | 0.209 |
| | -2.000 | 1.000 | 1.000 | 0.092 | 0.144 | 0.132 | 0.361 |
| 0.500 | 2.000 | 1.000 | 1.000 | 0.090 | 0.098 | 0.124 | 0.370 |
| | 1.000 | 1.000 | 1.000 | 0.094 | 0.091 | 0.108 | 0.188 |
| | 0.500 | 0.994 | 0.996 | 0.106 | 0.098 | 0.111 | 0.134 |
| | 0.000 | 0.124 | 0.117 | 0.096 | 0.110 | 0.097 | 0.101 |
| | -0.500 | 0.997 | 0.994 | 0.110 | 0.109 | 0.111 | 0.141 |
| | -1.000 | 1.000 | 1.000 | 0.082 | 0.096 | 0.108 | 0.190 |
| | -2.000 | 1.000 | 1.000 | 0.091 | 0.119 | 0.129 | 0.365 |
| 1.000 | 2.000 | 1.000 | 1.000 | 0.085 | 0.100 | 0.122 | 0.351 |
| | 1.000 | 1.000 | 1.000 | 0.101 | 0.115 | 0.118 | 0.191 |
| | 0.500 | 0.931 | 0.946 | 0.108 | 0.113 | 0.115 | 0.139 |
| | 0.000 | 0.115 | 0.122 | 0.093 | 0.098 | 0.092 | 0.095 |
| | -0.500 | 0.955 | 0.951 | 0.089 | 0.100 | 0.095 | 0.121 |
| | -1.000 | 1.000 | 1.000 | 0.094 | 0.122 | 0.113 | 0.196 |
| | -2.000 | 1.000 | 1.000 | 0.084 | 0.095 | 0.125 | 0.357 |

Table 2d The size of 10% nominal tests. Standard errors of agls and Linear IV
are consistent. N=1000, mc=1000, model is overidentified.

| Design | | Estimator | | | | | |
|----------|-----------|-----------|--------|-----------|-----------|-------|-------|
| θ | λ | ols | probit | IV probit | Linear IV | agls | tscml |
| 0.05 | 2 | 1.000 | 1.000 | 0.122 | 0.206 | 0.147 | 0.415 |
| | 1 | 1.000 | 1.000 | 0.108 | 0.133 | 0.117 | 0.184 |
| | 0.5 | 1.000 | 1.000 | 0.096 | 0.054 | 0.110 | 0.130 |
| | 0 | 0.086 | 0.084 | 0.099 | 0.023 | 0.100 | 0.099 |
| | -0.5 | 1.000 | 1.000 | 0.106 | 0.036 | 0.112 | 0.135 |
| | -1 | 1.000 | 1.000 | 0.085 | 0.090 | 0.115 | 0.195 |
| | -2 | 1.000 | 1.000 | 0.135 | 0.201 | 0.175 | 0.398 |
| 0.1 | 2 | 1.000 | 1.000 | 0.100 | 0.153 | 0.120 | 0.341 |
| | 1 | 1.000 | 1.000 | 0.091 | 0.138 | 0.123 | 0.199 |
| | 0.5 | 1.000 | 1.000 | 0.085 | 0.083 | 0.096 | 0.110 |
| | 0 | 0.111 | 0.109 | 0.109 | 0.065 | 0.109 | 0.109 |
| | -0.5 | 1.000 | 1.000 | 0.099 | 0.042 | 0.104 | 0.119 |
| | -1 | 1.000 | 1.000 | 0.093 | 0.076 | 0.131 | 0.192 |
| | -2 | 1.000 | 1.000 | 0.073 | 0.111 | 0.123 | 0.332 |
| 0.25 | 2 | 1.000 | 1.000 | 0.095 | 0.116 | 0.155 | 0.378 |
| | 1 | 1.000 | 1.000 | 0.098 | 0.108 | 0.126 | 0.201 |
| | 0.5 | 1.000 | 1.000 | 0.097 | 0.104 | 0.101 | 0.128 |
| | 0 | 0.102 | 0.109 | 0.095 | 0.100 | 0.095 | 0.095 |
| | -0.5 | 1.000 | 1.000 | 0.097 | 0.089 | 0.110 | 0.128 |
| | -1 | 1.000 | 1.000 | 0.108 | 0.112 | 0.125 | 0.207 |
| | -2 | 1.000 | 1.000 | 0.098 | 0.095 | 0.130 | 0.365 |
| 0.5 | 2 | 1.000 | 1.000 | 0.089 | 0.106 | 0.119 | 0.344 |
| | 1 | 1.000 | 1.000 | 0.085 | 0.104 | 0.107 | 0.179 |
| | 0.5 | 1.000 | 1.000 | 0.086 | 0.101 | 0.091 | 0.111 |
| | 0 | 0.089 | 0.093 | 0.109 | 0.106 | 0.106 | 0.108 |
| | -0.5 | 1.000 | 1.000 | 0.122 | 0.120 | 0.121 | 0.151 |
| | -1 | 1.000 | 1.000 | 0.087 | 0.095 | 0.112 | 0.195 |
| | -2 | 1.000 | 1.000 | 0.060 | 0.071 | 0.094 | 0.311 |
| 1 | 2 | 1.000 | 1.000 | 0.081 | 0.097 | 0.128 | 0.335 |
| | 1 | 1.000 | 1.000 | 0.095 | 0.108 | 0.116 | 0.187 |
| | 0.5 | 1.000 | 1.000 | 0.114 | 0.126 | 0.124 | 0.148 |
| | 0 | 0.103 | 0.107 | 0.122 | 0.117 | 0.120 | 0.121 |
| | -0.5 | 1.000 | 1.000 | 0.106 | 0.108 | 0.122 | 0.146 |
| | -1 | 1.000 | 1.000 | 0.088 | 0.102 | 0.114 | 0.201 |
| | -2 | 1.000 | 1.000 | 0.096 | 0.111 | 0.149 | 0.372 |

Table 3a Monte Carlo standard error each estimator based on samples of size 200, 1000 samples.
The model is just identified. The approximate proportion of 1's in each sample is .5.

| Design | | Estimator | | | | | | |
|----------|-----------|-----------|--------|-----------|-----------|-------|-------|---------|
| θ | λ | ols | probit | IV probit | Linear IV | agsl | tscml | pretest |
| 0.05 | 2 | 0.002 | 0.010 | 7.894 | 1.865 | 2.939 | 2.939 | 1.060 |
| | 1 | 0.002 | 0.005 | 2.063 | 0.715 | 1.086 | 1.086 | 0.712 |
| | 0.5 | 0.002 | 0.004 | 3.382 | 1.599 | 1.876 | 1.876 | 1.116 |
| | 0 | 0.002 | 0.003 | 12.405 | 7.046 | 8.544 | 8.544 | 0.378 |
| | -0.5 | 0.002 | 0.004 | 3.882 | 2.047 | 3.876 | 3.876 | 0.662 |
| | -1 | 0.002 | 0.005 | 1.773 | 1.389 | 3.186 | 3.186 | 0.434 |
| | -2 | 0.002 | 0.010 | 0.463 | 0.292 | 0.744 | 0.744 | 0.559 |
| 0.1 | 2 | 0.002 | 0.009 | 22.052 | 6.168 | 8.284 | 8.284 | 8.241 |
| | 1 | 0.002 | 0.005 | 3.107 | 0.440 | 0.918 | 0.918 | 0.646 |
| | 0.5 | 0.002 | 0.004 | 0.736 | 0.267 | 0.452 | 0.452 | 0.222 |
| | 0 | 0.002 | 0.003 | 12.608 | 7.070 | 8.960 | 8.960 | 0.108 |
| | -0.5 | 0.002 | 0.004 | 0.214 | 0.113 | 0.284 | 0.284 | 0.086 |
| | -1 | 0.002 | 0.005 | 0.755 | 0.551 | 1.002 | 1.002 | 0.981 |
| | -2 | 0.002 | 0.009 | 0.382 | 0.233 | 0.625 | 0.625 | 0.511 |
| 0.25 | 2 | 0.002 | 0.008 | 0.154 | 0.044 | 0.138 | 0.138 | 0.139 |
| | 1 | 0.002 | 0.005 | 0.075 | 0.028 | 0.050 | 0.050 | 0.052 |
| | 0.5 | 0.002 | 0.004 | 0.063 | 0.028 | 0.037 | 0.037 | 0.031 |
| | 0 | 0.002 | 0.003 | 0.064 | 0.027 | 0.045 | 0.045 | 0.033 |
| | -0.5 | 0.002 | 0.004 | 0.033 | 0.020 | 0.033 | 0.033 | 0.026 |
| | -1 | 0.002 | 0.005 | 0.057 | 0.043 | 0.085 | 0.085 | 0.087 |
| | -2 | 0.002 | 0.008 | 0.072 | 0.046 | 0.109 | 0.109 | 0.107 |
| 0.5 | 2 | 0.002 | 0.006 | 0.024 | 0.007 | 0.017 | 0.017 | 0.017 |
| | 1 | 0.002 | 0.004 | 0.018 | 0.006 | 0.011 | 0.011 | 0.012 |
| | 0.5 | 0.002 | 0.003 | 0.015 | 0.006 | 0.010 | 0.010 | 0.012 |
| | 0 | 0.002 | 0.003 | 0.012 | 0.006 | 0.009 | 0.009 | 0.006 |
| | -0.5 | 0.002 | 0.003 | 0.009 | 0.006 | 0.009 | 0.009 | 0.011 |
| | -1 | 0.002 | 0.004 | 0.008 | 0.006 | 0.011 | 0.011 | 0.012 |
| | -2 | 0.002 | 0.006 | 0.011 | 0.007 | 0.017 | 0.017 | 0.017 |
| 1 | 2 | 0.001 | 0.003 | 0.011 | 0.003 | 0.008 | 0.008 | 0.008 |
| | 1 | 0.002 | 0.003 | 0.008 | 0.003 | 0.005 | 0.005 | 0.005 |
| | 0.5 | 0.002 | 0.003 | 0.007 | 0.003 | 0.004 | 0.004 | 0.005 |
| | 0 | 0.002 | 0.003 | 0.006 | 0.003 | 0.004 | 0.004 | 0.003 |
| | -0.5 | 0.002 | 0.003 | 0.004 | 0.003 | 0.004 | 0.004 | 0.005 |
| | -1 | 0.002 | 0.003 | 0.004 | 0.003 | 0.005 | 0.005 | 0.005 |
| | -2 | 0.001 | 0.003 | 0.005 | 0.003 | 0.008 | 0.008 | 0.008 |

Table 3b Monte Carlo standard error each estimator based on samples of size 1000, 1000 samples.
 The model is just identified. The approximate proportion of 1's in each sample is .5.

| Design | | Estimator | | | | | | |
|----------|-----------|-----------|--------|-----------|-----------|--------|--------|---------|
| θ | λ | ols | probit | IV probit | Linear IV | agsl | tscml | pretest |
| 0.05 | 2 | 0.001 | 0.004 | 1.31 | 0.377 | 0.751 | 0.751 | 0.712 |
| | 1 | 0.001 | 0.002 | 0.821 | 0.297 | 0.49 | 0.49 | 0.304 |
| | 0.5 | 0.001 | 0.002 | 2.168 | 0.879 | 1.349 | 1.349 | 0.16 |
| | 0 | 0.001 | 0.001 | 2.438 | 1.193 | 1.724 | 1.724 | 1.551 |
| | -0.5 | 0.001 | 0.002 | 2.122 | 1.279 | 2.089 | 2.089 | 1.981 |
| | -1 | 0.001 | 0.002 | 8.888 | 6.092 | 11.608 | 11.608 | 11.607 |
| | -2 | 0.001 | 0.004 | 1.256 | 0.771 | 1.487 | 1.487 | 1.378 |
| 0.1 | 2 | 0.001 | 0.004 | 0.368 | 0.1 | 0.243 | 0.243 | 0.243 |
| | 1 | 0.001 | 0.002 | 3.428 | 1.253 | 1.714 | 1.714 | 0.056 |
| | 0.5 | 0.001 | 0.002 | 0.682 | 0.297 | 0.401 | 0.401 | 0.053 |
| | 0 | 0.001 | 0.001 | 0.195 | 0.099 | 0.138 | 0.138 | 0.129 |
| | -0.5 | 0.001 | 0.002 | 0.207 | 0.123 | 0.222 | 0.222 | 0.204 |
| | -1 | 0.001 | 0.002 | 0.038 | 0.029 | 0.051 | 0.051 | 0.049 |
| | -2 | 0.001 | 0.004 | 0.501 | 0.311 | 0.623 | 0.623 | 0.623 |
| 0.25 | 2 | 0.001 | 0.003 | 0.02 | 0.006 | 0.014 | 0.014 | 0.014 |
| | 1 | 0.001 | 0.002 | 0.015 | 0.005 | 0.009 | 0.009 | 0.01 |
| | 0.5 | 0.001 | 0.002 | 0.013 | 0.005 | 0.008 | 0.008 | 0.01 |
| | 0 | 0.001 | 0.001 | 0.01 | 0.005 | 0.007 | 0.007 | 0.005 |
| | -0.5 | 0.001 | 0.002 | 0.008 | 0.005 | 0.008 | 0.008 | 0.01 |
| | -1 | 0.001 | 0.002 | 0.007 | 0.005 | 0.009 | 0.009 | 0.009 |
| | -2 | 0.001 | 0.003 | 0.009 | 0.006 | 0.014 | 0.014 | 0.014 |
| 0.5 | 2 | 0.001 | 0.003 | 0.01 | 0.003 | 0.007 | 0.007 | 0.007 |
| | 1 | 0.001 | 0.002 | 0.007 | 0.003 | 0.004 | 0.004 | 0.004 |
| | 0.5 | 0.001 | 0.001 | 0.006 | 0.003 | 0.004 | 0.004 | 0.004 |
| | 0 | 0.001 | 0.001 | 0.005 | 0.002 | 0.004 | 0.004 | 0.003 |
| | -0.5 | 0.001 | 0.001 | 0.004 | 0.003 | 0.004 | 0.004 | 0.004 |
| | -1 | 0.001 | 0.002 | 0.003 | 0.003 | 0.004 | 0.004 | 0.004 |
| | -2 | 0.001 | 0.002 | 0.004 | 0.003 | 0.006 | 0.006 | 0.006 |
| 1 | 2 | 0.001 | 0.001 | 0.005 | 0.001 | 0.003 | 0.003 | 0.003 |
| | 1 | 0.001 | 0.001 | 0.003 | 0.001 | 0.002 | 0.002 | 0.002 |
| | 0.5 | 0.001 | 0.001 | 0.003 | 0.001 | 0.002 | 0.002 | 0.002 |
| | 0 | 0.001 | 0.001 | 0.002 | 0.001 | 0.002 | 0.002 | 0.001 |
| | -0.5 | 0.001 | 0.001 | 0.002 | 0.001 | 0.002 | 0.002 | 0.002 |
| | -1 | 0.001 | 0.001 | 0.002 | 0.001 | 0.002 | 0.002 | 0.002 |
| | -2 | 0.001 | 0.001 | 0.002 | 0.001 | 0.003 | 0.003 | 0.003 |

Table 4a Comparison of agls and LIML. Sample size = 200, model just identified.
 Upper panel compars the coefficient on the endogenous variable ($\gamma=0$)
 Lower panel compares the percentiles to the pvalue of the corresponding t-ratio.

| λ | θ | 0.5 | | 2 | | 0.5 | | 2 | |
|--|-----------|---------|----------|----------|----------|--------|--------|--------|--------|
| | | 0.1 | | 0.1 | | 1 | | 1 | |
| | | agls | LIML | agls | LIML | agls | LIML | agls | LIML |
| C o e f e c i e n t | 1% | -44.751 | -1.021 | -45.860 | -0.96689 | -0.563 | -0.371 | -0.720 | -0.325 |
| | 5% | -7.270 | -0.947 | -10.488 | -0.85039 | -0.347 | -0.271 | -0.425 | -0.235 |
| | 10% | -3.649 | -0.864 | -5.034 | -0.70906 | -0.271 | -0.221 | -0.328 | -0.195 |
| | 25% | -0.790 | -0.489 | -0.842 | -0.27075 | -0.137 | -0.118 | -0.173 | -0.114 |
| | 50% | 0.300 | 0.293 | 1.117 | 0.888625 | -0.008 | -0.008 | -0.009 | -0.006 |
| | 75% | 1.462 | 1.003 | 2.994 | 1.557343 | 0.113 | 0.109 | 0.136 | 0.108 |
| | 90% | 3.645 | 1.111 | 8.057 | 2.068173 | 0.221 | 0.219 | 0.246 | 0.212 |
| | 95% | 8.198 | 1.166 | 12.735 | 2.246212 | 0.270 | 0.269 | 0.318 | 0.272 |
| | 99% | 48.105 | 1.253 | 64.591 | 2.512663 | 0.420 | 0.417 | 0.433 | 0.384 |
| | Mean | -0.368 | 0.235 | 3.462 | 0.703199 | -0.020 | -0.005 | -0.029 | 0.001 |
| Variance Skewness Kurtosis | Std. Dev. | 31.512 | 0.756 | 87.029 | 1.033331 | 0.193 | 0.167 | 0.233 | 0.158 |
| | Variance | 992.991 | 0.571 | 7574.060 | 1.067773 | 0.037 | 0.028 | 0.055 | 0.025 |
| | Skewness | -10.139 | -0.216 | 19.665 | -0.0193 | -0.341 | 0.155 | -0.502 | 0.395 |
| | Kurtosis | 255.376 | 1.546 | 497.026 | 1.71487 | 3.670 | 3.050 | 3.758 | 3.495 |
| p - v a l u e s | 1% | 0.077 | 0.00E+00 | 0.004 | 7.46E-17 | 0.019 | 0.001 | 0.017 | 0.004 |
| | 5% | 0.222 | 1.78E-38 | 0.037 | 1.33E-06 | 0.079 | 0.027 | 0.075 | 0.045 |
| | 10% | 0.299 | 2.60E-16 | 0.105 | 0.001 | 0.129 | 0.083 | 0.126 | 0.097 |
| | 25% | 0.479 | 3.92E-04 | 0.329 | 0.076 | 0.265 | 0.228 | 0.277 | 0.245 |
| | 50% | 0.697 | 0.222 | 0.660 | 0.393 | 0.517 | 0.517 | 0.499 | 0.489 |
| | 75% | 0.868 | 0.696 | 0.856 | 0.720 | 0.773 | 0.775 | 0.753 | 0.755 |
| | 90% | 0.952 | 0.915 | 0.934 | 0.884 | 0.905 | 0.905 | 0.903 | 0.903 |
| | 95% | 0.976 | 0.958 | 0.965 | 0.938 | 0.957 | 0.958 | 0.954 | 0.954 |
| | 99% | 0.996 | 0.995 | 0.994 | 0.987 | 0.995 | 0.995 | 0.984 | 0.983 |

Table 4b Comparison of agls and LIML. Sample size = 1000, model just identified.
 Upper panel compars the coefficient on the endogenous variable ($\gamma=0$)
 Lower panel compares the percentiles to the pvalue of the corresponding t-ratio.

| λ | θ | 0.5 | | 2 | | 0.5 | | 2 | |
|---|-----------|--------|----------|--------|--------|--------|--------|--------|--------|
| | | 0.25 | | 0.25 | | 1 | | 1 | |
| | | agls | LIML | agls | LIML | agls | LIML | agls | LIML |
| C o e f f e c i e n t | 1% | -1.379 | -0.646 | -2.295 | -0.548 | -0.222 | -0.183 | -0.261 | -0.160 |
| | 5% | -0.709 | -0.454 | -1.212 | -0.370 | -0.154 | -0.133 | -0.168 | -0.109 |
| | 10% | -0.532 | -0.376 | -0.901 | -0.307 | -0.115 | -0.104 | -0.128 | -0.086 |
| | 25% | -0.247 | -0.199 | -0.439 | -0.177 | -0.060 | -0.054 | -0.074 | -0.050 |
| | 50% | -0.013 | -0.012 | -0.006 | -0.003 | -0.005 | -0.005 | -0.001 | 0.000 |
| | 75% | 0.218 | 0.210 | 0.338 | 0.187 | 0.051 | 0.049 | 0.063 | 0.048 |
| | 90% | 0.411 | 0.410 | 0.601 | 0.388 | 0.102 | 0.099 | 0.125 | 0.096 |
| | 95% | 0.534 | 0.533 | 0.736 | 0.505 | 0.130 | 0.128 | 0.158 | 0.127 |
| | 99% | 0.787 | 0.748 | 0.961 | 0.731 | 0.201 | 0.199 | 0.220 | 0.177 |
| | Mean | -0.042 | 0.009 | -0.101 | 0.021 | -0.005 | -0.002 | -0.004 | 0.002 |
| | Std. Dev. | 0.397 | 0.300 | 0.643 | 0.273 | 0.087 | 0.080 | 0.100 | 0.072 |
| p - v a l u e s | Variance | 0.158 | 0.090 | 0.414 | 0.075 | 0.007 | 0.006 | 0.010 | 0.005 |
| | Skewness | -0.845 | 0.257 | -1.243 | 0.455 | -0.104 | 0.112 | -0.141 | 0.210 |
| | Kurtosis | 5.384 | 2.832 | 6.080 | 3.172 | 3.182 | 3.099 | 2.937 | 2.877 |
| | 1% | 0.010 | 7.38E-05 | 0.004 | 0.004 | 0.006 | 0.003 | 0.009 | 0.008 |
| | 5% | 0.069 | 0.006 | 0.050 | 0.050 | 0.040 | 0.031 | 0.042 | 0.042 |
| | 10% | 0.114 | 0.037 | 0.129 | 0.108 | 0.090 | 0.079 | 0.094 | 0.091 |
| | 25% | 0.255 | 0.215 | 0.288 | 0.261 | 0.232 | 0.234 | 0.245 | 0.236 |
| | 50% | 0.506 | 0.498 | 0.509 | 0.494 | 0.505 | 0.501 | 0.488 | 0.484 |
| | 75% | 0.757 | 0.760 | 0.736 | 0.734 | 0.753 | 0.754 | 0.724 | 0.724 |
| | 90% | 0.907 | 0.907 | 0.896 | 0.895 | 0.910 | 0.910 | 0.886 | 0.887 |
| | 95% | 0.959 | 0.959 | 0.946 | 0.946 | 0.955 | 0.955 | 0.941 | 0.941 |
| | 99% | 0.995 | 0.995 | 0.989 | 0.989 | 0.988 | 0.988 | 0.992 | 0.992 |